Skill-Biased Technological Change, Inequality, and the Role of Retraining*

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Abstract

The collapse of routine occupations driven by skill-biased technological change has shrunk economic opportunities for less-educated workers. Retraining could provide the workers displaced by occupational decline with opportunities to gain skills that growing occupations require. In this paper, I study the equilibrium effects of retraining in an economy with directed job search. Not only does retraining improve participants' skills, it also changes non-participants' optimal job search strategies and, in turn, their re-employment outcomes. I find that retraining reduces between-skill inequality whereas it increases within-skill inequality. Eliminating retraining makes everyone worse off, causing losses equivalent to a 1.5 percent drop in consumption. I also evaluate various labor market policies that aim to encourage retraining participation. I show that combining retraining with a more generous unemployment insurance benefit is the most cost-effective policy. It also results in the highest average welfare.

JEL Classification: E24, J24, J64, J68

Keywords: Retraining, Skill-biased technological change, Directed job search model, Income inequality

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1 Introduction

It has been well documented that the share of employment in middle-wage routine occupations has declined in the U.S. labor market in large part due to automation and so-called Skill-Biased Technological Change (Autor et al., 2008; Acemoglu and Autor, 2011; David and Dorn, 2013). This trend has worsened re-employment prospects for the unemployed who previously held such occupations. They are more likely to fall into long-term unemployment, leave the labor force, or shift into low-wage service occupations (Lee and Wolpin, 2010; Cortes et al., 2014, 2017). Retraining could help unemployed workers to obtain jobs in growing occupations and sectors that usually require greater cognitive skills. Despite its potential role, retraining hasn't received as much attention as other policy responses to unemployment, such as unemployment insurance benefit or search assistance. To fill this gap, in this paper I study retraining in an economy with directed job search and use it to investigate the equilibrium effects of retraining on wage and employment.

I first document a set of novel stylized facts about retraining. Using the National Longitudinal Survey of Youth for both 1979 and 1997, I show how prevalent retraining is among unemployed workers, what affects unemployed workers' decisions to retrain, and whether it improves their career prospects. I find that (a) retraining rates among unemployed workers aged 23-34 in the NLSY79 are only around 2 percent, but the rates increase by about 5 percentage points in the NLSY97; (b) sex, race, marital status, learning ability, and asset holdings affect unemployed workers' participation in retraining; (c) the completion rate of retraining is low; and (d) conditional on completing retraining, retraining participants are more likely to get better-paying jobs that involve less routine, manual tasks and more non-routine, cognitive tasks.

Based on this evidence, I build a model with directed job search and retraining. I consider an economy comprised of two occupation groups (cognitive and routine), two types of workers (high- and low-skill) who are heterogeneous in several dimensions, and frictional labor markets. The model has two important features. First, low-skill unemployed workers are given a chance to upgrade their skills by participating in retraining. Retraining corresponds to college attendance since most training for unemployed workers in the U.S. takes this form (Jacobson et al., 2005a). Retraining entails an opportunity cost as well as monetary costs since participants need to forgo labor market activities while retraining. The completion of retraining is assumed to be stochastic, reflecting high college dropout rates among non-conventional students. Successful completion of retraining ensures workers higher wages and a higher probability that they leave the routine occupation for the non-routine cognitive occupation. As observed in the data, unemployed workers' age and wealth are primary determinants of retraining participation. Second, the labor market features directed search. As in Menzio and Shi (2010), Menzio et al. (2016), Eeckhout and Sepahsalari (2014), and Herkenhoff et al. (2016), unemployed workers decide which job to apply for. The labor market consists of multiple submarkets distinguished by workers' age, skill, and occupation. In each submarket, there is a continuum of firms that offer various wages. In equilibrium, there is an inverse relationship between wages and job finding rates. High-paying jobs are more difficult to obtain. Unemployed workers choose which job to apply for by comparing wages against the probability of employment. Workers' wealth is a crucial factor in their optimal search strategies. Workers with high asset levels choose to wait until they are matched with a high-paying job because they can endure prolonged unemployment by relying on their savings. On the contrary, low-asset workers choose to apply for low-paying jobs so that they can get out of unemployment as quickly as possible.

Retraining affects the wage distribution through workers' job search strategies, asset holdings, and the income tax rate. The opportunity to retrain increases the value of unemployment by expanding unemployed workers' choice sets. Higher value of unemployment induces workers to make bolder choices when they apply for jobs. They choose to apply for higher-paying jobs at a given asset level. Consequently, wages increase for low-skill workers. Meanwhile, retraining decreases participants' asset holdings. Due to the monetary and opportunity costs of retraining, the participants are likely to hold low levels of assets. That makes them apply for low-paying jobs, offsetting some of the positive effects of retraining on wages. Moreover, as more workers end up at the lower end of the wage distribution, the variance of wages increase. Lastly, retraining increases after-tax wages by reducing the income tax rate. The equilibrium income tax rate is determined as the ratio of the government spending on unemployment insurance benefit to the income tax revenue. As more high-skill workers are created through retraining, the income tax revenue increases, and therefore, the income tax rate decreases.

The model is calibrated to the U.S. economy to the NLSY79. The calibrated model matches the mean retraining rate by age well. The model also does a good job of generating the rise in retraining between the NLSY79 and NLSY97 cohorts. To see this, I adjust a set of parameters that capture the changes in the labor market that the NLSY97 cohort experienced such as the increase in wage premium, the decrease in job finding rates, and the increase in job separation rates. The model explains around 79 percent of the increase in retraining participation observed in the data. The changes in job transition rates for low-skill workers play a bigger role than the changes in wage premium do, implying that grim prospects in the labor market for low-skill workers are the most important motive to retrain.

Using the model, I make three quantitative contributions. First, I study the effects of retraining on wage inequality. Since the collapse of middle-wage jobs is considered a primary source of rising inequality, it is important to understand how retraining affects it. To this end, I compare the benchmark economy with retraining to a counterfactual economy where retraining is not possible. I find that in the economy where unemployed workers have a retraining option, low-skill workers go for higher-paying jobs and as a result, earn higher wages. This decreases the wage gap between low- and high-skill workers. However, within-skill inequality measured by wage variance is larger in the benchmark economy for both low- and high-skill workers. Newly-created high-skill workers tend to have low asset levels since they ran down their savings while retraining. To avoid extended unemployment, they apply for low-paying jobs, making the wage distribution more dispersed. Similarly, the variance of wages among low-skill workers is higher in the benchmark economy since the participants who fail to complete retraining end up with low-paying jobs.

Second, I investigate the welfare effects of retraining. I assume that the workers in the benchmark economy are transferred to an economy without retraining and calculate the welfare changes. Moving to the economy without retraining makes everyone worse-off. It causes a 1.5% drop in the average welfare. For high-skill workers, welfare losses come from exclusively from the income tax increase. For low-skill workers, the income tax increase explains 38% of the total losses. The losses also come from changes in optimal job search strategies and savings. With a lack of retraining, low-skill workers take safe job search strategies. Facing a low probability of unemployment, they are in less need of precautionary savings. It alleviates some of the total losses. The lost opportunities of upgrade skill accounts for the remaining welfare losses for low-skill workers.

Lastly, I suggest several government policies that can encourage retraining participation and evaluate the effectiveness of each policy by comparing their effects on retraining rates and required tax increases. Universal free retraining results in the highest retraining participation. However, it comes with a high tax increase. I find that guaranteeing higher unemployment insurance benefit for retraining participants achieves the biggest increase in retraining rates for a given tax increase. It is the policy that maximizes the average welfare as well.

This paper is related to several strands of literature. First, a number of studies such as Meyer (1995), Heckman et al. (1999), Jacobson et al. (2005a), Jacobson et al. (2011), Nie (2010), and Barr and Turner (2015) investigate the determinants and consequences of job-training and education programs for unemployed workers. These studies tend to conduct individual-level analysis focusing on the effects of retraining on individual re-employment outcomes. Nie (2010) is the only exception. He develops a structural framework of retraining and uses it to examine macroeconomic effects of retraining. Specifically, he shows how reforms of retraining programs in Germany affect aggregate employment, unemployment, and output. This paper is different from Nie (2010) in that, by incorporating directed search in the model, it takes account of the effect retraining has on non-participants as well, which allows for a more general welfare and policy analysis.

From the model perspective, this study is indebted to a growing literature of directed job search models such as Menzio and Shi (2010, 2011), Menzio et al. (2016), Eeckhout and Sepahsalari (2014), and Herkenhoff et al. (2016). I contribute to this literature by applying the theory of directed job search to an important social issue of retraining low-skill workers. The directed job search framework has been used in examining the effects of passive labor market policies such as unemployment insurance benefits (Acemoglu and Shimer, 1999; Chaumont and Shi, 2017). To my knowledge, this is the first attempt to use this framework to analyze active labor market policies such as retraining. Lastly, it is also related to an extensive literature on skill-biased technological change and job polarization. This paper extends this literature by assessing the potential role of retraining in mitigating the negative consequences of the decline in middle-wage occupations.

This paper is structured as follows: Section 2 describes the data and empirical results. Section 3 presents the model. Section 4 discusses the calibration strategy and the quantitative analysis, and Section 5 provides the conclusion.

2 Empirical Evidence

In this section, I present a number of new stylized facts about retraining that will motivate the setup of my model. Mainly, I look at (a) how prevalent retraining is in the U.S. (b) what characteristics retraining participants have and, (c) how retraining changes participants' re-employment wages and occupations later.

2.1 Data and Sample Selection

My empirical results are based on both the NLSY79 and NLSY97. The NLSY79 and NLSY97 are longitudinal studies that follow American youth born between 1957-1964 and between 1980-1984, respectively. Using both NLSY surveys together provides two advantages. First, since the NLSY97 only has information on relatively young population, also using the NLSY79 allows me to observe retraining patterns for older population. Second, since the NLSY79 survey was made before the decline in the routine occupation had started whereas the NLSY97 survey was made when it had been ongoing for a while, comparing the two surveys' population provides some insight into how the decline of routine occupation has impacted retraining participation. Unlike the NLSY79, the NLSY97 doesn't include the economically disadvantaged non-black, non-Hispanic oversample and the military sample. To make the population being studied comparable across two surveys, I exclude these extra samples from the NLSY79.

I first restrict the sample to those who ever experienced at least one Employment-Unemployment-Employment transition. To focus on the impact retraining has on low-skill workers, I exclude those who had a college degree at the beginning of the unemployment spell. I then define retraining participants as those who enrolled in a 2- or 4-year college during the unemployment spell. The panel structure of the NLSY allows me to find the exact time that a person lost his job and the time that he started college. A person is considered to have participated in retraining if he enrolled in a 2- or 4-year college after he had lost his job and before he had found a new one. The way I define retraining participants is in line with the fact that much of retraining programs for unemployed workers in the U.S. takes place in the form of regular college courses (Jacobson et al., 2005a).[1] Besides, looking at college programs rather than particular job training programs is most suited for the purpose of the study since most highly demanded jobs require a college degree.

One concern about the way I define retraining participants is that with the information available in the NLSY surveys, it is difficult to perfectly distinguish retraining participants from those who are simply putting off going to college. However, this problem doesn't seem critical. The youngest group in the sample is 23 years old, the age at which most people finish college education. Plus, the workers in the sample have the average of about 5 years of full-time working experience prior to unemployment. So, it's reasonable to assume that they are different than those who take one or two gap years for some experience before starting college.

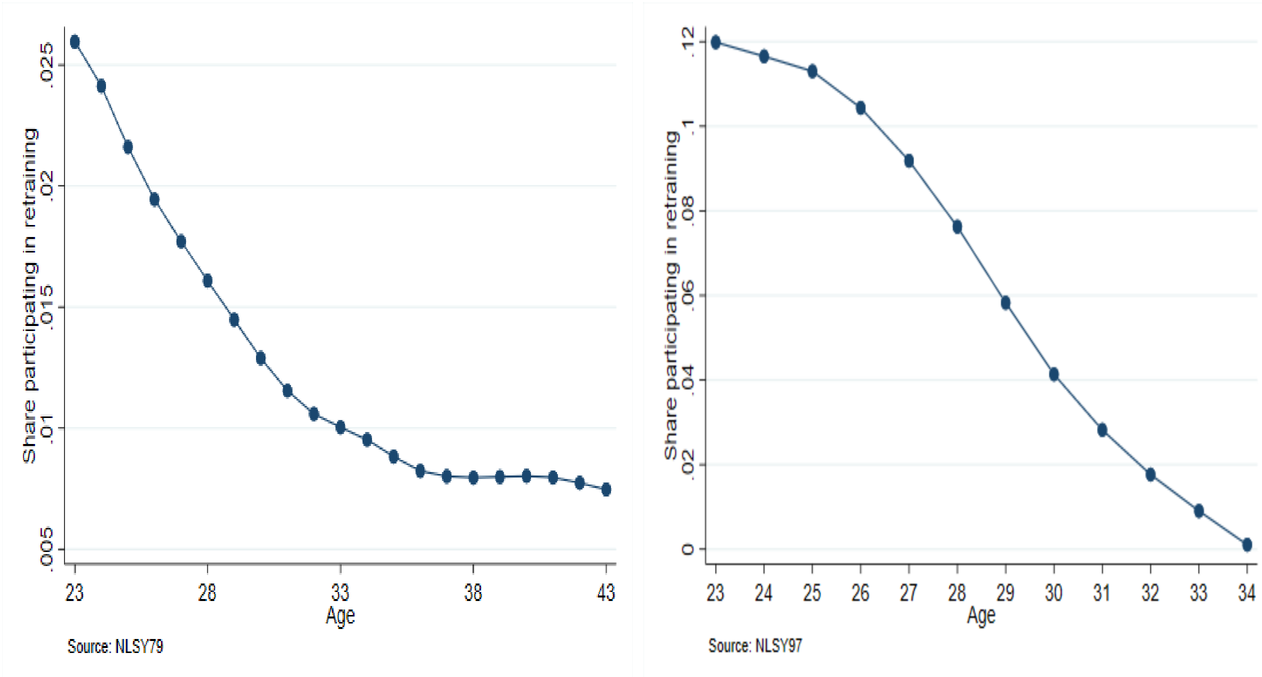
The data is at the unemployment spell level. There are a total of 11,981 spells for a total of 4,347 individuals in the NLSY79, and a total of 5,050 spells for 2,610 individuals in the NLSY97.

2.2 Incidence of retraining

Figure 1 plots the fraction of retraining participants among the unemployed by age. The top panel plots the results from the NLSY79, and the bottom panel from the NLSY97. Retraining rates decrease along age, which is not surprising since older workers have fewer working years left to enjoy the rewards of retraining. The figure also shows that there has been a big increase

[1] Public-sponsored retraining in the U.S. for unemployed workers is provided through the Workforce Investment Act. Unemployed workers can receive three tiers of services: core services (e.g. job search assistance), intensive services (e.g. comprehensive assessment, case management), and training (e.g. classroom training, on-the-job training). Workers who reach the training level of services are given a voucher referred to as Individual Training Accounts, which they can use to obtain retraining from certified providers, most of which are 2-year public colleges. (Eberts, 2010; Frank and Minoff, 2005)

Figure 1: Retraining rates by age



Note: This figure presents the share of retraining participants among the unemployed at each age. The left figure is from the NLSY79 and the right figure the NLSY97.

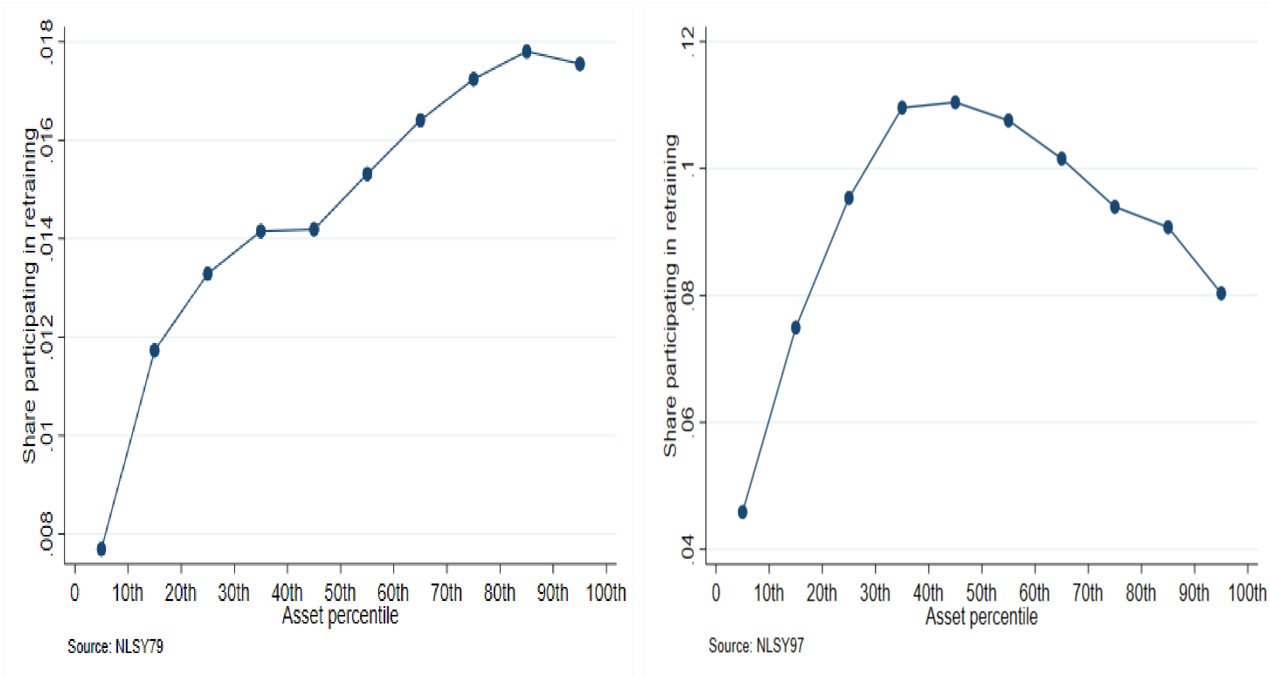
in retraining for the younger cohorts. At the age of 23, only 2.2% of unemployed workers retrained in the NLSY79. The number increases to 10.8% among the NLSY97 cohorts.[2] I discuss the sources of this increase in the later part of the paper.

2.3 Characteristics of retraining participants

Table 1 compares basic summary statistics between retraining participants and non-participants. Retraining participants are slightly younger, more likely to be female, more likely to be black, less likely to be Hispanic, and less likely to be married. These findings are consistent with what Barr and Turner (2015) found from the CPS. Retraining participants also have a higher

[2]To my knowledge, Barr and Turner (2015) is the only recent study on prevalence of retraining. Using the CPS, they found 13 % of unemployed individuals aged 20-30 were enrolled in college between 2008 and 2011, and from the SIPP, they found between 15 and 20% of UI recipients aged 20-30 enrolled within 6 months of initial UI receipt over a similar period. Their numbers are bigger than what I found possibly because the definition of retraining participants I used is more restrictive.

Figure 2: Retraining rates by asset percentile



Note: This figure plots retraining rates over the residual asset distribution. The left figure is from the NLSY79 and the right figure the NLSY97.

cognitive ability measured by Armed Forces Qualifications Test score (AFQT).

I found no significant differences between the two groups in the net value of total assets they hold and hourly wages they used to earn pre-unemployment. Since there are more young people, females, and minorities among retraining participants, I compared asset holdings and previous wages adjusted for the effects of age, sex, race, marital status, and AFQT scores. The results show that retraining participants have higher residual assets. The difference is significant at the 10% significance level. Figure 2 gives a closer look on the relation between asset holdings and retraining. The figure plots retraining rates over the distribution of residual total assets. The fraction of retraining participants among the unemployed increases as assets percentile increases. In the high end of the asset distribution, however, the fraction flattens then slightly decreases.

Unemployed workers' previous occupations can affect their retraining participation as well. Those who previously worked in the occupations where higher-education is rewarded more

Table 1: Descriptive statistics (Participants vs. Non-participants)

	All		Non participants		Participants		
Variable	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.	
Age	26.41	2.72	26.51	2.77	25.44	2.04	***
Percent male	45.88	0.50	46.70	0.50	37.85	0.49	***
Percent black	28.99	0.45	28.64	0.45	32.47	0.47	**
Percent Hispanic	22.00	0.41	22.36	0.42	18.49	0.39	**
Percent married	25.43	0.44	25.80	0.44	21.72	0.41	*
Total real asset ($)	24509.21	37408.13	24473.15	36865.67	24864.89	42429.83	
Residual total asset($)	2.04E-05	36916.82	-274.25	36378.16	2704.17	41806.26	*
Real hourly wages($)	13.22	25.77	13.21	25.82	13.36	25.29	
Residual wages($)	-8.34E-09	26.35	-0.06	25.78	0.63	25.23	
Percent non-routine cognitive	25.52	0.44	25.43	0.44	26.45	0.44	
Percent non-routine manual	21.41	0.41	20.94	0.41	26.02	0.44	**
Percent routine cognitive	34.85	0.48	34.94	0.48	33.98	0.47	
Percent routine manual	18.22	0.39	18.69	0.39	13.55	0.34	***
AFQT	43679.24	26345.29	43096.80	26282.72	49142.68	26347.08	***
N of Obs	5,050		4,585		465		

Note: *Significant at 10%, **Significant at 5%, ***Significant at 1%
The corresponding table for the NLSY79 can be found in Table B1.
Source: NLSY97.

(e.g. non-routine cognitive occupation) may have a stronger incentive to retrain. On the other hand, those who worked in the occupations in decline (e.g. routine occupation) may want to retrain more so that they can move to the occupations in expansion. Following the literature (Acemoglu and Autor, 2011; David and Dorn, 2013; Cortes et al., 2014, 2017), I classify workers' previous occupations into four groups: non-routine cognitive, non-routine manual, routine cognitive, routine manual[3]. I then compare the share of each occupation group between retraining participants and non-participants. Since the share of women, who have a higher participation rate than men, varies across occupations, I do this analysis separately by sex. The results are reported in Table 2. For women, I find no significant differences in previous occupations between retraining participants and non-participants. For men, however, retraining participants have a higher fraction of former non-routine manual workers and a lower fraction of former routine-manual workers. This suggests that low retraining participation among men stems from low participation among former routine-manual workers, despite the fact that it is the occupation most vulnerable to automation and international trade and, therefore, its workers need retraining the most.

2.4 Outcomes of retraining

In this section, I examine the outcomes of participating in retraining. Before doing this, I first see how many retraining participants successfully complete retraining. I compute the fraction of retraining participants who earn either an Associate's degree or a Bachelor's degree by years from the beginning of the unemployment spell. The success rate of retraining is quite low. As shown in Table 3, after 4 years from the start of unemployment, only about 33 percent of retraining participants hold an Associate's or Bachelor's degree. After 6 years, the number increases to about 42 percent. Only just less than a half were able to get a college degree as a result of retraining. This finding is consistent with that non-traditional students who are

[3]The occupation is considered routine if the tasks can be done by following well-defined instructions. The occupation is considered non-routine if it involves tasks that require flexibility, creativity, problem-solving, or human interaction. Cognitive and manual occupations are distinguished by the relative extent of mental to physical activity. Non-routine cognitive occupations include Professional, Managerial and Technical Occupations. Routine cognitive occupations include Sales and Clerical Occupations. Routine manual occupations include Production, Craft and Repair Occupations, Operators, and Transportation and Material Moving Occupations. Non-routine manual occupations include Service Occupations.(Cortes et al., 2017)

Table 2: Occupation by sex (Participants vs. Non-participants)

A. Male	Non-participants	Participants	
Percent non-routine cognitive	19.6	19.3	
Percent non-routine manual	21.0	29.5	***
Percent routine cognitive	27.0	26.7	
Percent routine manual	32.3	24.4	**
B. Female	**Non-participants**	**Participants**	
Percent non-routine cognitive	30.5	30.8	
Percent non-routine manual	20.9	23.9	
Percent routine cognitive	41.9	38.4	
Percent routine manual	6.8	6.9	

Note: *Significant at 10%, **Significant at 5%, ***Significant at 1%
The corresponding table for the NLSY79 can be found in Table B2.
Source: NLSY97.

Table 3: Retraining completion rates

	Percent holding a college degree
t+4	32.92
t+5	38.14
t+6	42.24

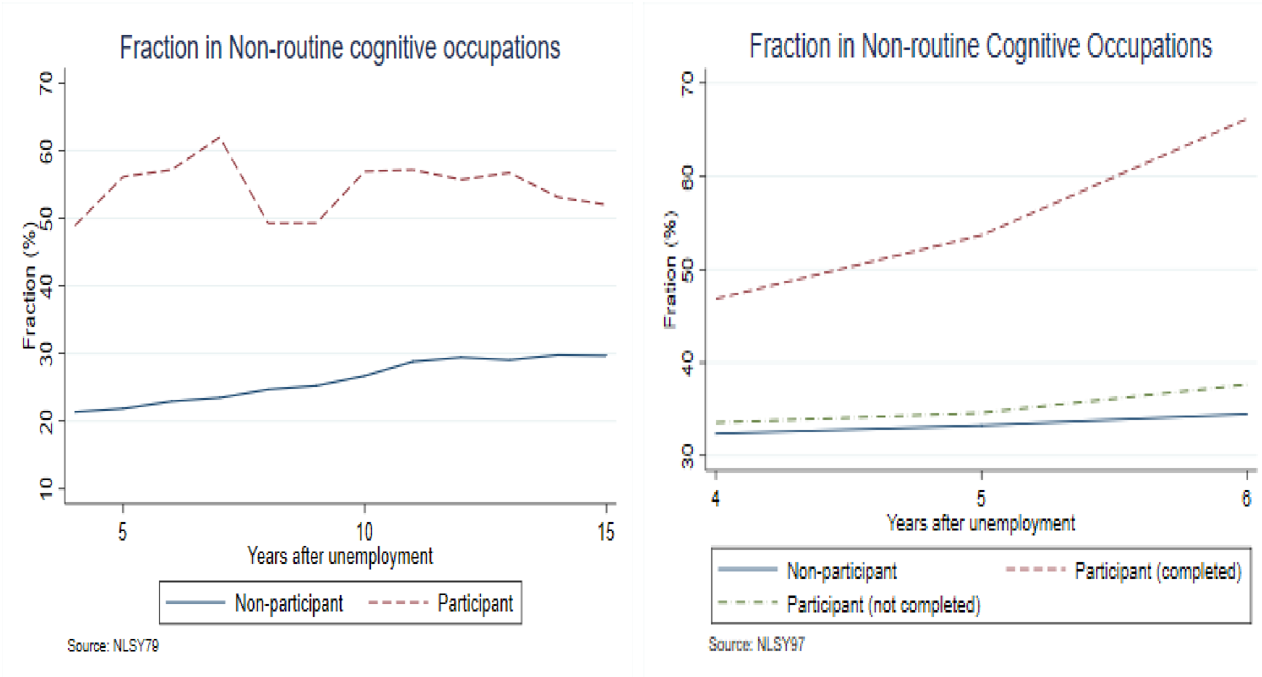
Note: t is the time when a sample lost his job
The corresponding table for the NLSY79 can be found in Table B2.
Source: NLSY97.

usually older than traditional students have higher college dropout rates than others.

2.4.1 Occupation Switching Patterns

I begin by comparing the occupation switching patterns among unemployed workers. I see if participating in retraining affects the probability of a worker moving to a higher-ranked job in the job ladder. Again, I use the occupation classification of Acemoglu and Autor (2011). Ranked by occupational mean wage, non-routine cognitive occupations are in the top, non-routine manual occupations are in the bottom, and routine occupations are in the middle. Figures 3-5 show the share of workers re-employed in each occupation group. The share of workers re-employed in the non-routine cognitive occupation, the highest ranked group, is higher for participants than non-participants. The share re-employed in the routine occupation, which

Figure 3: Fraction in non-routine cognitive occupations

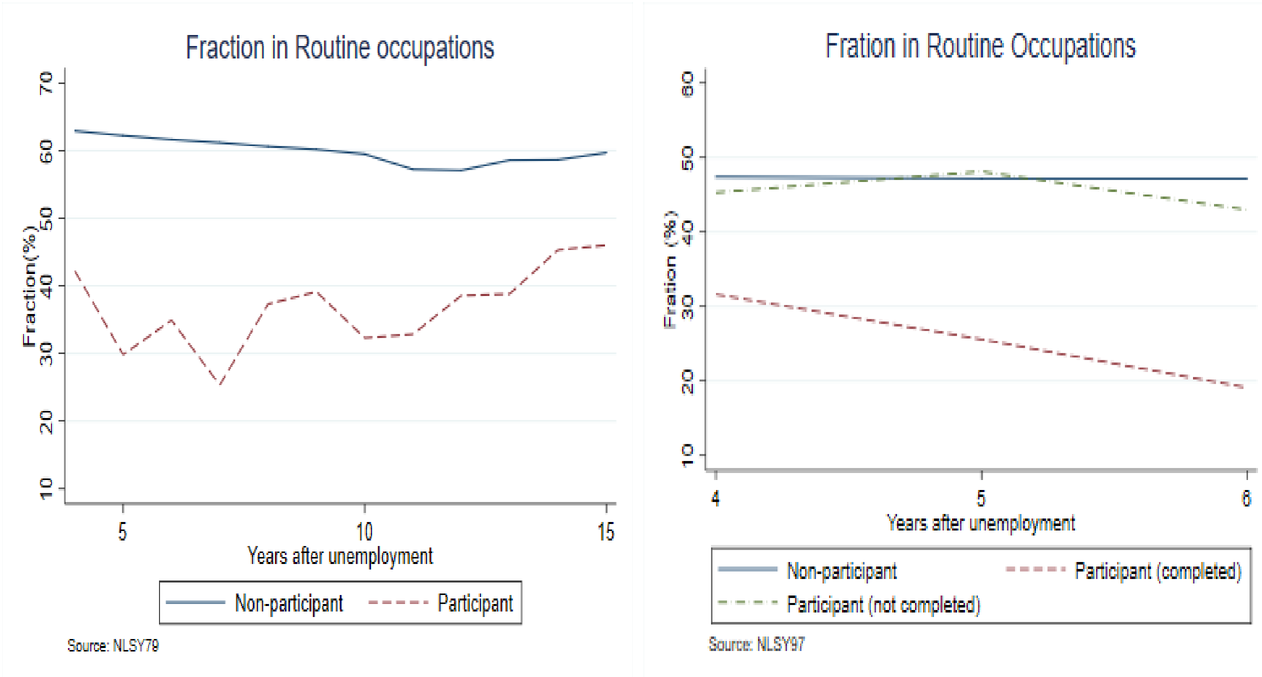


Note: This figure presents the share of workers who work in non-routine cognitive occupations. The horizontal axis shows years after the job loss.

has been in decline for the past couple of decades, is lower among the participants. The share in the non-routine manual occupation, the lowest ranked group, is also lower for participants. However, the results for retraining participants who fail to get a college degree aren't very different from those for non-participants.

I also classify the sample according to the direction of the switches. The results are presented in Figures 6-8. The share of workers who switched to higher-ranked occupations (e.g. from routine to non-routine cognitive, from non-routine manual to routine/non-routine cognitive) is higher for participants. Both the share of stayers and the share of those who moved down the ladder (e.g from non-routine cognitive to routine/non-routine manual, from routine to non-routine manual) are lower among participants. In the NLSY97, the fraction moving down is actually higher for participants at the time they would have just finished retraining, but it decreases as time passes and becomes lower than non-participants eventually. Overall, retraining participants have a better chance of finding a better job when they are re-employed than

Figure 4: Fraction in routine occupations



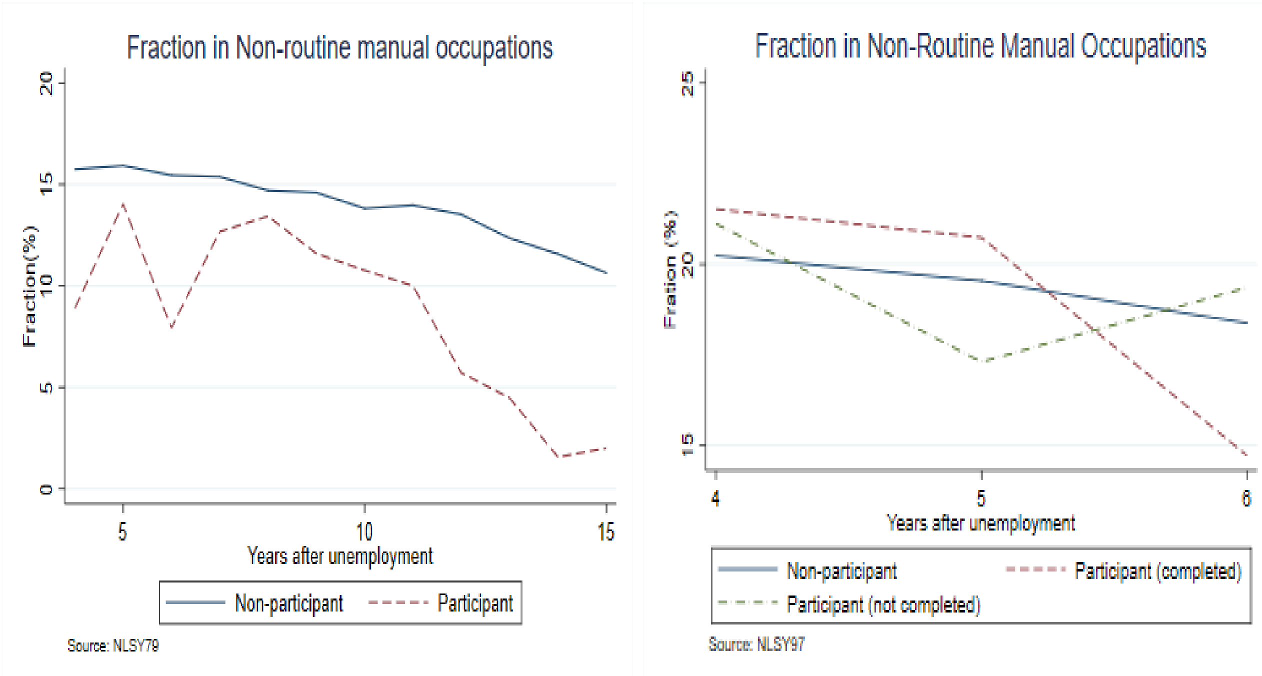
Note: This figure presents the share of workers who work in routine occupations. The horizontal axis shows years after the job loss.

non-participants. It is not the case, though, for those who stop retraining without a degree.

2.4.2 Wage changes

Now I turn my interest to the effects of retraining on wage changes. Figure 9 compares wage changes between participants and non-participants. After 4 years from the beginning of unemployment, wages of retraining participants are only slightly higher than those of non-participants. This is closely related to the fact that the share of individuals who switch occupations is higher among participants. Those who switch occupations tend to start with low wages since they have to start over in a new field where they don't have much experience. Plus, while participants are studying at school, non-participants can keep working, which increase their tenure and in turn, their wages. However, as I showed in the previous section, retraining participants are more likely to work in the non-routine cognitive occupation where the average wage grows more quickly. This is reflected in their wage changes. Wages of participants increase

Figure 5: Fraction in non-routine manual workers



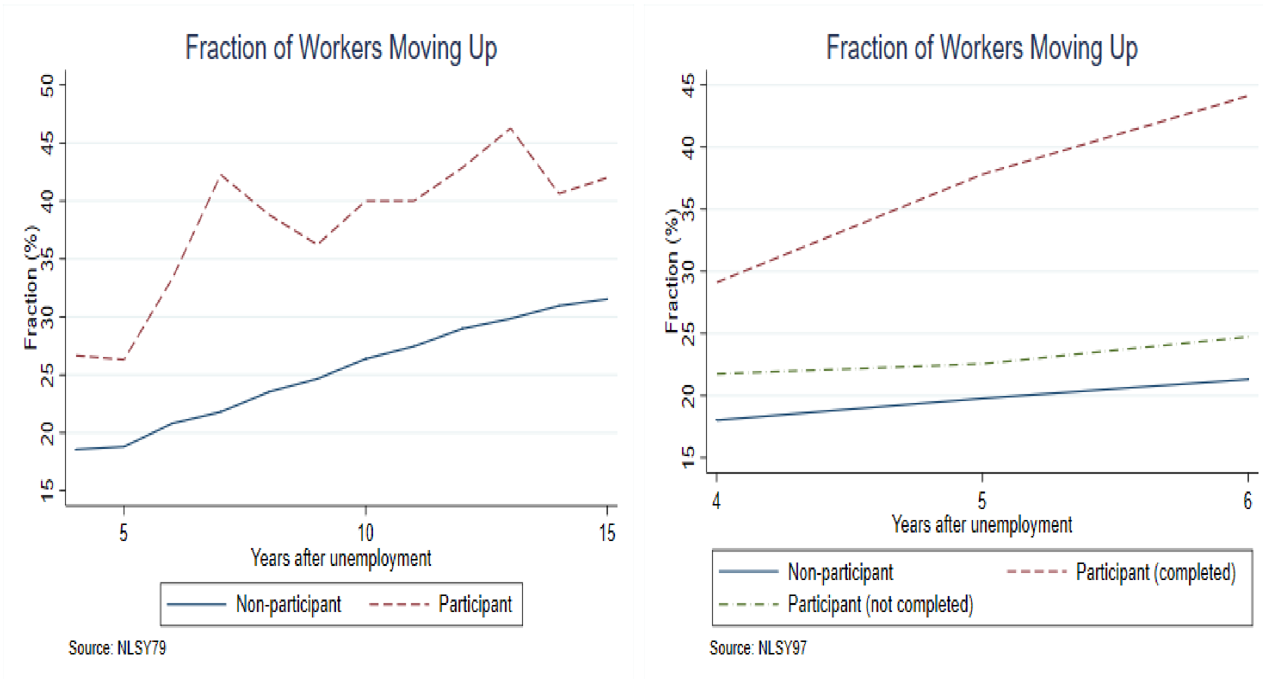
Note: This figure presents the share of workers who work in non-routine manual occupations. The horizontal axis shows years after the job loss.

faster with time, making the gap between participants and non-participants wider. Comparing the NLSY79 and 97, wage changes are biggest among recent cohorts, reflecting increased skill and occupation premium.

3 Model

Time is discrete and lasts forever. There is a unit measure of risk-averse finitely-lived workers. Each worker lives $T \geq 2$ periods deterministically, thus there are T overlapping generations in the economy. A worker's utility in each period is $u(c) + L_\epsilon \eta + \psi \mathbb{1}_{\{\text{retraining}=1\}}$. c is consumption. The function u satisfies $u'(c) \geq 0$ and $u''(c) < 0$. η is the utility from leisure where ϵ denotes the worker's employment status. ψ is workers' preference for studying. This preference parameter captures factors not explicitly modeled here that may affect their retraining participation, such as their learning ability. Workers discount the future at a rate $\beta \in (0, 1)$ and accumulate non-

Figure 6: Fraction of workers who moved up the job ladder



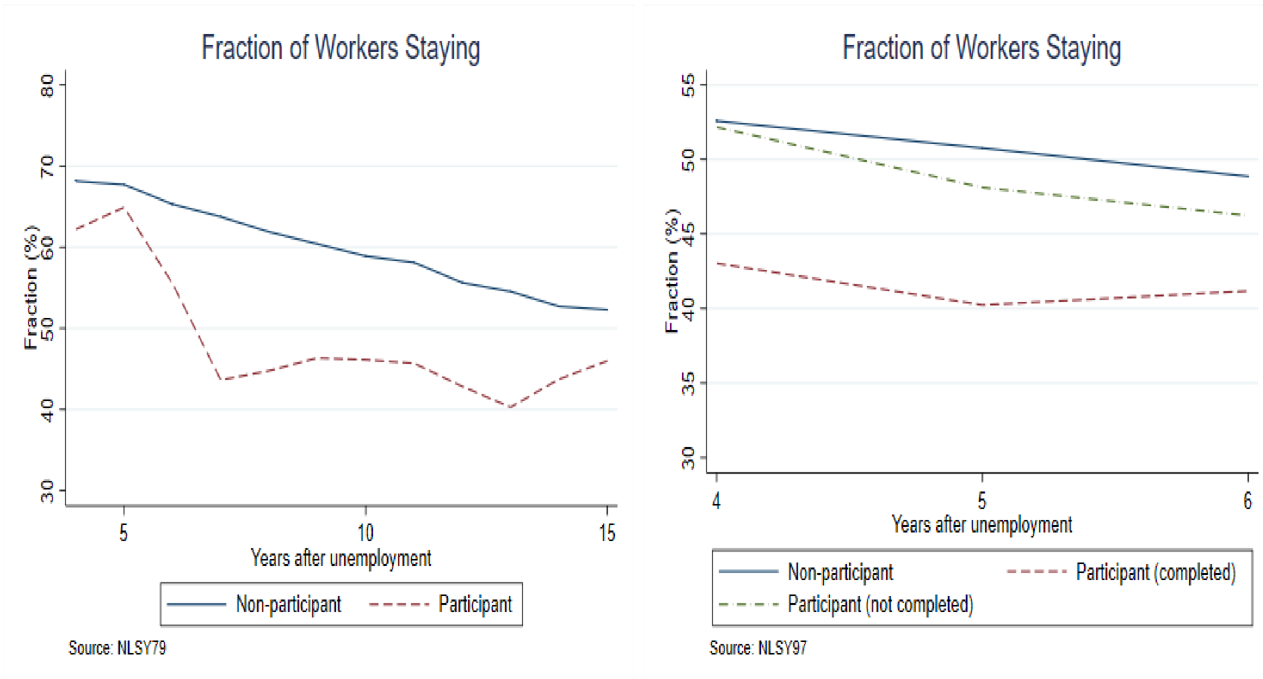
Note: This figure presents the share of workers who moved up the job ladder (e.g., workers who switched from routine occupations to non-routine cognitive occupations). The horizontal axis shows years after the job loss.

contingent assets denoted as $a \in A = [\underline{a}, \overline{a}] \subset R$. The net rate of return on assets r is taken as given.

Workers are born with skill $s \in \{l, h\}$. Workers born with h represent those who enter the labor market with a college degree, whereas workers born with l represent those who start their careers with only a high-school diploma. In each period, workers with skill s are either employed or unemployed, where the employed value function is denoted E^s and unemployed value function is denoted U^s. Employed workers spend $1 - L_e$ amount of time working and receive wage w each period. They pay a fraction of their wage τw as income tax. Unemployed workers spend $1 - L_u$ searching for jobs and receive unemployment insurance benefit $b > 0$, which expires every period with the probability χ. Once they lose their benefit, they can't receive it again during the same unemployment spell.

There is a continuum of risk neutral firms. Firms belong to either the routine occupation

Figure 7: Fraction of workers who stayed in the same occupation group

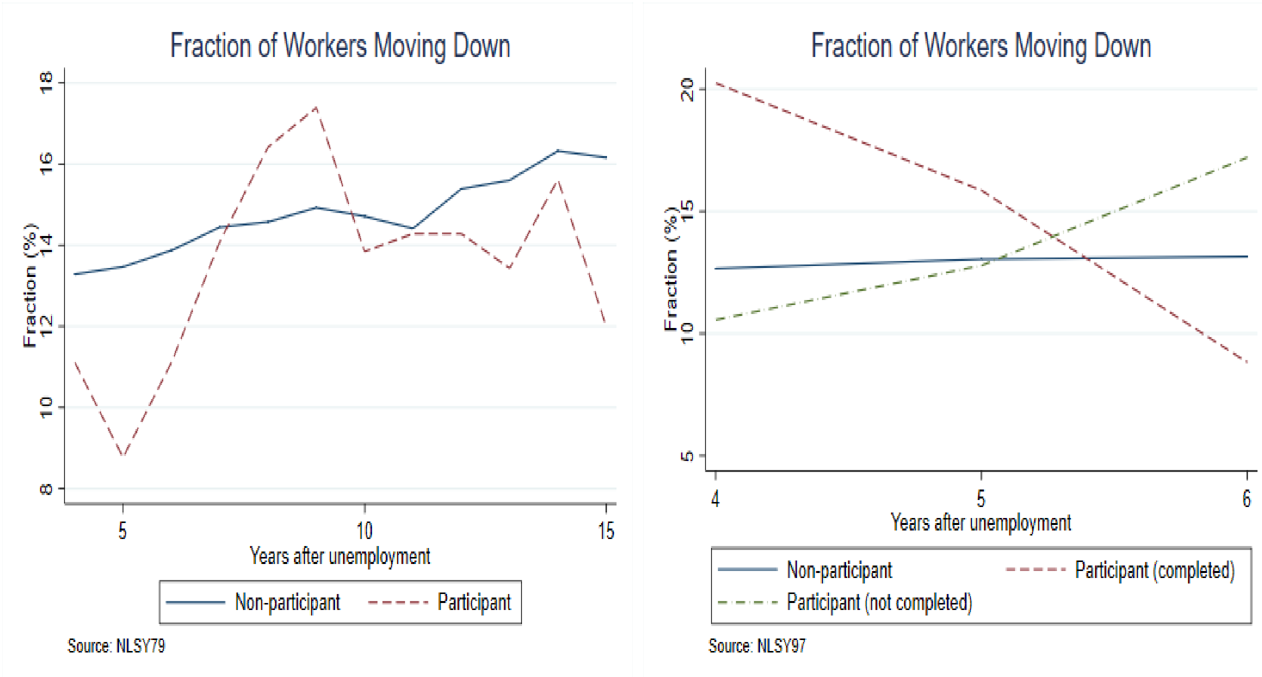


Note: This figure presents the share of workers who went back to the occupation they had held prior to unemployment. The horizontal axis shows years after the job loss.

group (denoted by R) or the non-routine cognitive occupation group (denoted by CG). Matching between a skill-s worker and a occupation-j firm produces y_{js} units of output. I assume skill-h workers produce more than skill-l workers and occupation-CG firms produce more than occupation-R firms. Therefore, $y_{CG,h} > y_{R,h} > y_{CG,l} > y_{R,l}$. Matching between skill and occupation is determined exogenously. Skill-s workers meet with occupation-CG firms with the probability p_s. Skill-h workers have a higher chance to be matched with occupation-CG firms than skill-l workers ($p_h > p_l$). Each period, firms post a vacancy at cost κ_{js}. When posting a vacancy, firms offer a contract that specifies the piece-rate of output $\mu \in [0, 1]$ that is paid as wages. A contract is not renegotiable, fixing μ until the match breaks. For simplicity, I assume there is no on-the-job search. The only way to break an existing match is exogenous separation, which happens every period with the occupation and skill-specific probability δ_{js}.

The labor market consists of a continuum of submarkets indexed by worker age t, skill s, occupation j, and the piece-rate μ. Each submarket (t, s, j, μ) has its own tightness $\theta_t(s, j, \mu)$,

Figure 8: Fraction of workers who moved down the job ladders



Note: This figure presents the share of workers who moved down the job ladder (e.g., workers who switched from non-routine cognitive occupations to routine occupations). The horizontal axis shows years after the job loss.

which is defined as the ratio of vacancies to job applicants. The matching process in each submarket is governed by a constant returns to scale matching function $M(u(t,s,j,\mu), v(t,s,j,\mu))$. Workers' job finding rates are defined as:

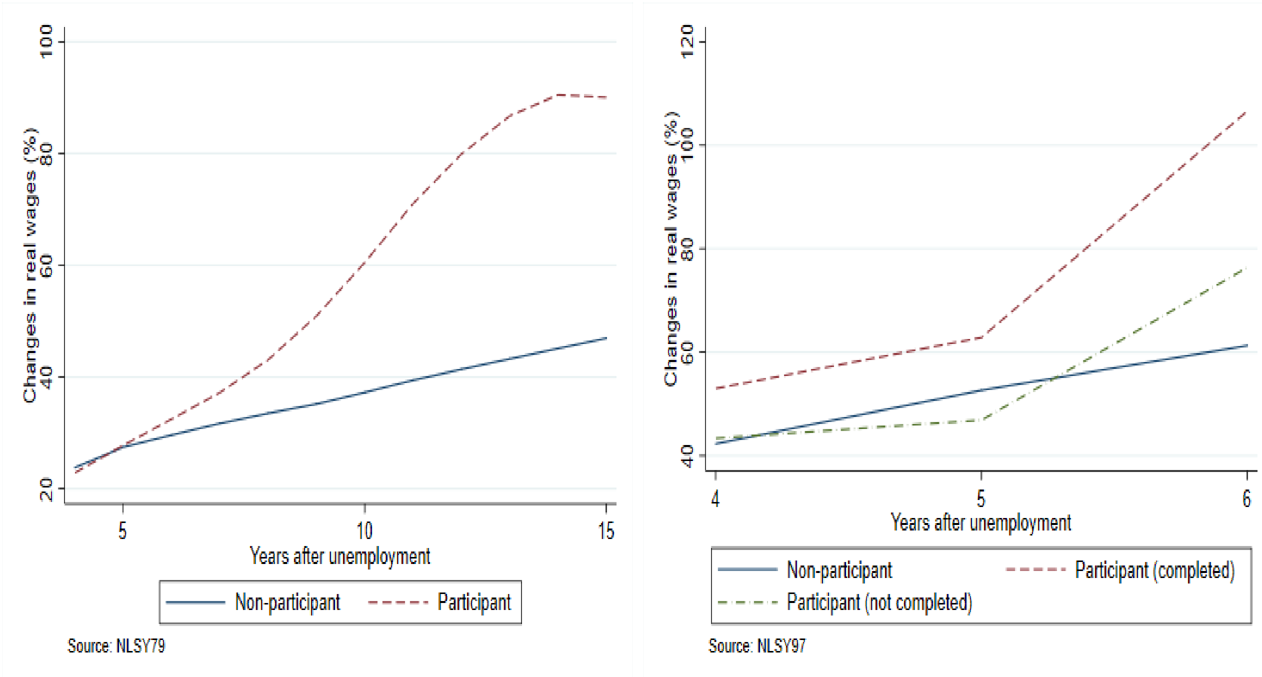
$$m(\theta_t(s,j,\mu)) = \frac{M(u(t,s,j,\mu), v(t,s,j,\mu))}{u(t,s,j,\mu)}$$

As $\theta_t(s,j,\mu)$ increases, it becomes easier for workers to find the employer, thus $m'(\theta_t(s,j,\mu)) > 0$. Firms' hiring rates are given as:

$$q(\theta_t(s,j,\mu)) = \frac{M(u(t,s,j,\mu), v(t,s,j,\mu))}{v(t,s,j,\mu)}$$

It becomes harder for firms to find the employee as θ increases, therefore $q'(\theta_t(s,j,\mu)) < 0$. In each labor market, the free entry condition determines the measure of firms. In each period, unemployed workers choose the submarket in which they search for jobs by comparing the wage

Figure 9: Wage changes



Note: This figure compares wage changes between retraining participants and non-participants. The vertical axis presents changes in real wages compared to workers' most recent wages prior to unemployment. The horizontal axis shows years after the job loss.

$w(\theta_t(s, j, \mu)) = \mu y_{js}$ and the probability they get hired $m(\theta_t(s, j, \mu))$.

Low-skill workers can upgrade their skills through retraining. If a skill-l worker is participating in retraining, his value function is scripted with a R, and if not, scripted with a NR. At the beginning of each period, skill-l unemployed workers decide whether they participate in retraining or not by comparing $U^{l,NR}$ and $U^{l,R}$. If they decide to participate, they spend $1-L_r$ amount of time in retraining for $\bar{z}$ periods. Retraining requires financial and opportunity costs. Participants pay tuition every period, and they are not allowed to work while retraining. I assume participants are still eligible for insurance benefits.[4] The retraining process is stochastic. In each period, participants face a dropout risk. They stop retraining with the probability of $\lambda \in [0, 1]$. Once completing retraining without dropping out, they become skill-h workers, in which case they experience higher wages and higher chances to be matched with a

[4]In the U.S., unemployment insurance beneficiaries are allowed to enroll in college or job skills training while also receiving benefits as long as they enroll in approved programs (Barr and Turner (2015).)

occupation-CG firm.

The government provides unemployed workers with insurance benefits and subsidizes their retraining. ρ fraction of tuition ν is paid by the government. The government finances the retraining costs and insurance benefits by imposing income taxes on employed workers.

In each period, the timeline is given as follows: (1) Existing matches produce and workers are paid. Unemployed workers receive their insurance benefit. (2) Unemployed workers decide whether to participate in retraining or not. (3) Workers choose optimal consumption and saving. (4) Unemployed workers who are not participating retraining choose the submarket in which they search for jobs, and new matches are created. They don't start to produce until the next period. (5) A fraction δ_j of existing matches is separated. Newly formed matches are excluded in this exogenous separation process. (6) A fraction λ of trainees drop out. (7) A fraction χ of unemployed workers lose their unemployment insurance benefit.

3.1 Unemployed workers

In this section, I describe the problem of low-skill unemployed workers. Low-skill unemployed workers decide whether they participate in retraining or search for a job at the beginning of each period. They face a different problem according to their decisions. I describe retraining participants' and non-participants' problems, and unemployed workers' retraining decisions.

3.1.1 Retraining participants

The problem of retraining participants is given below:

$$\begin{aligned} U_t^{l,R}(a,b,\psi,j,z) = \max_{a'} u(c, L_r, \psi) &+ \lambda\beta[\chi U_{t+1}^{l,NR}(a',0,\psi,j) + (1-\chi)U_{t+1}^{l,NR}(a',b,\psi,j)] \\ &+ (1-\lambda)\beta[\chi U_{t+1}^{l,R}(a',0,\psi,j,z') + (1-\chi)U_{t+1}^{l,R}(a',b,\psi,j,z')], \quad t \leq T \quad (1) \end{aligned}$$

$$U_{T+1}^{l,R}(a,b,\psi,j,z) = 0$$

$$\text{s.t} \quad c + a' + (1-\rho)\nu = (1+r)a + b$$
$$a' \geq \underline{a}$$

$z = [z_1, z_2, \cdots, \overline{z}]$ denotes semesters. At each period, retraining participants choose optimal consumption and saving (c and a'), receive insurance benefit b, and pay tuition $(1-\rho)\nu$ where ρ is the share of tuition paid by the government. Their current utility depends on consumption c, leisure L_r and their preference for studying ψ. In the next period, they drop out with the probability of λ, in which case they fail to upgrade their skills. It is not allowed that retraining participants come back to school at the same period they drop out. With the probability of $1-\lambda$, they go on to the next semester z' and continue retraining.

Each period, χ fraction of benefit recipients lose their benefit. The problem of those who have already lost their benefit is given below:

$$U_t^{l,R}(a, 0, \psi, j, z) = \max_{a'} u(c, L_r, \psi) + \lambda\beta U_{t+1}^{l,NR}(a', 0, \psi, j) + (1-\lambda)\beta U_{t+1}^{l,R}(a', 0, \psi, j, z'), \quad t \leq T \quad (2)$$

$$U_{T+1}^{l,R}(a, 0, \psi, j, z) = 0$$

$$\text{s.t} \quad c + a' + (1-\rho)\nu = (1+r)a + b_{min}$$
$$a' \geq \underline{a}$$

where b_{min} is home production that prevents negative consumption.

The value function of retraining participants takes a different form in the last semester $(z = \overline{z})$. The problem is given below:

$$U_t^{l,R}(a, b, \psi, j, \overline{z}) = \max_{a'} u(c, L_r, \psi) + \lambda\beta[\chi U_{t+1}^{l,NR}(a', 0, \psi, j) + (1-\chi)U_{t+1}^{l,NR}(a', b, \psi, j)] + (1-\lambda)\beta\chi[p_h U_{t+1}^{h}(a', 0, CG) + (1-p_h)U_{t+1}^{h}(a', 0, R)] + (1-\lambda)\beta(1-\chi)[p_h U_{t+1}^{h}(a', b, CG) + (1-p_h)U_{t+1}^{h}(a', b, R)], \quad t \leq T \quad (3)$$

$$U_{T+1}^{l,R}(a,b,\psi,j,\overline{z}) = 0$$

$$\begin{aligned} \text{s.t} \quad & c + a' + (1-\rho)\nu = (1+r)a + b \\ & a' \geq \underline{a} \end{aligned}$$

Retraining participants leave school and go on to the labor market in the next period. But with the probability of λ, they fail to complete retraining and search for jobs as low-skilled in the occupation group to which they belonged before they had started retraining. With the probability of $1-\lambda$, they finish retraining with a degree and search for jobs as high-skilled. Those who successfully complete retraining can either search in occupation-CG or occupation-R. It is not guaranteed for them to go into sector-CG, but they have a higher chance to do that. p_h denotes the probability that a skill-h worker searches in occupation-CG.

3.1.2 Non-participants

The value function of skill-l unemployed workers who are not participating in retraining is given as:

$$\begin{aligned} U_t^{l,NR}(a,b,\psi,j) = \max_{a'} u(c,L_u) + \chi\beta[\max_{\mu'} m(\theta_{t+1}(l,j,\mu'))E_{t+1}^l(a',\mu',\psi,j) \\ + (1-m(\theta_{t+1}(l,j,\mu')))U_{t+1}^l(a',0,\psi,j)] + (1-\chi)\beta[\max_{\mu'} m(\theta_{t+1}(l,j,\mu'))E_{t+1}^l(a',\mu',\psi,j) \\ + (1-m(\theta_{t+1}(l,j,\mu')))U_{t+1}^l(a',b,\psi,j)], \quad t \leq T \end{aligned} \tag{4}$$

$$U_{T+1}^{l,NR}(a,b,\psi,j) = 0$$

$$\text{s.t} \quad c + a' = (1+r)a + b \quad \text{and} \quad a' \geq \underline{a}$$

Their utility depends on consumption c and leisure L_u. Besides optimal savings a', workers choose labor markets in which they search for jobs. Since j is determined exogenously, they only choose μ', the contract between a firm and a worker on what fraction of production the worker takes. Choosing μ', they are hired with the probability of $m(\theta(l,j,\mu'))$, in which case, they get paid $w(\theta(l,j,\mu'))$. As mentioned earlier, there exists an inverse relation between $m(\theta(l,j,\mu'))$ and $w(\theta(l,j,\mu'))$ at the equilibrium.

Skill-l unemployed workers make a retraining decision at the beginning of every period:

$$U_t^l(a,b,\psi,j) = \max\left\{U_t^{l,R}(a,b,\psi,j,z_1), U_t^{l,NR}(a,b,\psi,j)\right\} \tag{5}$$

Let $D_t(a,b,\psi,j)$ denote the worker's retraining decision. $D_t(a,b,\psi,j) = 1$ when the value of retraining is larger than the value of staying low-skilled ($U_t^{l,R}(a,b,\psi,j,z_1) > U_t^{l,NR}(a,b,\psi,j)$).

3.2 Employed workers

The value function of employed workers is given below:

$$E_t^l(a,\mu,\psi,j) = \max_{a'} u(c,L_e) + \beta[\delta_{js}U_{t+1}^l(a',b,\psi,j) + (1-\delta_{js})E_{t+1}^l(a',\mu,\psi,j)] \tag{6}$$

$$E_{T+1}^l(a,\mu,\psi,j) = 0$$

$$\text{s.t} \quad c + a' = (1+r)a + (1-\tau)w(\theta_t(l,j,\mu)) \quad \text{and} \quad a' \geq \underline{a}$$

Employed workers' current utility depends on consumption c and leisure L_e. They choose optimal consumption c and saving a', get paid labor income $w(\theta_t(l,j,\mu))$, and pay income tax $\tau w(\theta_t(l,j,\mu))$. In the next period, with probability δ_{js}, they separate from the firm they are currently working for. For simplicity, I assume there is no on-the-job search, and employed workers don't participate in retraining.

The value functions for skill-h workers are included in the Appendix.

3.3 Firms

In each labor market (t,s,j,μ), there's a continuum of firms. Each firm hires a single worker. Firms post a vacancy with a contract that specifies a piece-rate μ of production they pay to their workers. Contracts are renegotiation-proof. An occupation-j firm that hires a skill-s worker produces y_{js} units of output, which represent the matching quality between the firm and the worker. The firm retains a fraction $(1-\mu)$ of the output and pays the rest to the worker. There's no on-the-job search, but the match can break exogenously. The probability that a match between skill-s workers and occupation-j firms exogenously breaks is δ_{js} The value function for firms is given as:

$$J_t(s,j,\mu) = (1-\mu)y_{js} + \beta(1-\delta_{js})J_{t+1}(s,j,\mu), \quad t \leq T \tag{7}$$

$$J_{T+1}(s,j,\mu) = 0$$

The free entry condition holds for each submarket (t,s,j,μ). The occupation and skill-specific cost of posting a vacancy, κ_{js}, is equal to the expected benefit of posting a vacancy. This yields:

$$\kappa_{js} = q(\theta_t(s,j,\mu))J_t(s,j,\mu) \tag{8}$$

In equilibrium, equation (7) and (8) together yield the market tightness in each submarket:

$$\theta_t(s,j,\mu) = q^{-1}\left(\frac{\kappa_{js}}{J_t(s,j,\mu)}\right) \tag{9}$$

3.4 Equilibrium

An equilibrium in this economy is a set of policy functions for workers $\{c, a', \mu', D\}$, value functions for workers $U_t^s, U_t^{l,NR}, U_t^{l,R}, E_t^s$, value functions for firms J_t, a market tightness function $\theta_t(s,j,\mu)$, an income tax rate τ, and the economy's density function f. These functions satisfy the following:

1. The policy functions solve the workers problems with associated value functions.
2. The free entry condition holds.
3. The total income tax revenue equals the summation of the total amount of unemployment insurance benefit and tuition subsidy
4. The distribution of workers across state is consistent with workers' policy functions.

Table 4: Independently chosen model parameters

Parameter	Value	Description	Source
T	140	Life span	Standard
r	0.012	Risk free rate	Annual rate $\approx$ 5%
β	0.988	Discount factor	1/(1+r)
σ	2	Risk aversion	Standard
$\underline{a}$	-2	Debt limit	Non-binding borrowing constraint
η	0.237	Flow utility of leisure	Herkenhoff et al. (2016)
L_e	0.875	Time spent working	Albanesi and Sahin (2018)
L_u	0.375	Time spent job searching	Albanesi and Sahin (2018)
ζ	0.5	Matching efficiency	Shi (2016)
$\delta_{CG,h}$	0.02	Separation rate at occupation-CG for skill-h	CPS (1983-39)
$\delta_{CG,l}$	0.034	Separation rate at occupation-CG for skill-l	CPS (1983-39)
$\delta_{R,h}$	0.034	Separation rate at occupation-R for skill-h	CPS (1983-39)
$\delta_{R,l}$	0.061	Separation rate at occupation-R for skill-l	CPS (1983-39)
λ	0.08	College dropout rate	NLSY97
p_h	0.7347	Prob that a type-h works at the NRCG occupation	CPS (1983-89)
p_l	0.2183	Prob that a type-l works at the NRCG occupation	CPS (1983-89)
M_h	0.2508	Fraction born as type-h	CPS (1983-89)
b	0.32	UI benefit	Benefit income ratio $\approx$ 40%
χ	0.788	UI benefit expiration rate	Expected UI duration $\approx$ 26 weeks

4 Quantitative Analysis

4.1 Calibration

In this section, I discuss the parameterization of the model. I divide the model parameters into three groups. For the first set of parameters, I either borrow values from other literature or use standard values. The values of the second set of parameters are chosen directly to match their counterparts in the data. The third set of parameters are jointly calibrated to the U.S. data, to the cohort born 1957-1964. A list of parameters included in each group are summarized in Table 4 and 5.

The length of a period is calibrated to a quarter, and the model age zero corresponds to age 18 in the data. The workers leave the model at the model age of 140. All workers enter the model unemployed and with zero assets. I assume a quarterly interest rate equal to 1.2%, which yields an annual rate of 5%. Workers are born as either skill-l or skill-h. The fraction born as skill-h, M_h, is set to the share of college graduates at age 23 in the Current Population

Table 5: Jointly-calibrated parameters

Parameter	Value	Description
$y_{CG,h}$	1.39	Matching quality between occupation-CG and skill-h
$y_{R,h}$	1.156	Matching quality between occupation-R and skill-h
$y_{CG,l}$	1.128	Matching quality between occupation-CG and skill-l
$y_{R,l}$	1	Matching quality between occupation-R and skill-l
$\kappa_{CG,h}$	0.6481	Vacancy posting cost at occupation-CG for skill-h
$\kappa_{R,h}$	0.3104	Vacancy posting cost at occupation-R for skill-h
$\kappa_{CG,l}$	0.3932	Vacancy posting cost at occupation-CG for skill-l
$\kappa_{R,l}$	0.2772	Vacancy posting cost at occupation-R for skill-l
ψ_{μ}	0.2701	Scale parameter in the preference distribution
ν	0.092	Tuition

Survey (CPS). Unemployed workers search for jobs either in occupation-CG or occupation-R. The fraction of skill-s workers who search in occupation-CG, p_s, is chosen to match the share of each skill type (college graduates or high-school graduates) in the non-routine cognitive occupation calculated from the CPS.

Preferences for workers at a given period are given below:

$$u(c, L_{\epsilon}, \psi) = \frac{c^{1-\sigma} - 1}{1-\sigma} + \eta(1 - L_{\epsilon}) + \psi \mathbb{1}_{\{\text{retraining}=1\}}, \quad \text{where} \quad \epsilon = e, r, u \tag{10}$$

The discount factor, β, is set to 0.988 so that $\beta = 1/(1+r)$. The risk aversion parameter, σ, is set to a standard value, 2.

The utility from leisure, η, is set to 0.237 following Herkenhoff et al. (2016). L_e is set to 0.625 (10 hours of work out of 16 active hours) and L_u to 0.125 (2 hours of job searching for jobs out of 16 active hours) following Albanesi and Şahin (2018) and Krueger and Mueller (2012). I assume that retraining participants spend as much time at school as employed workers spend at work. Thus, $L_r = L_e$.

The utility from studying, ψ, is 6 evenly spaced grid points over [0.6,1.4]. Low-skill workers are born with a draw over this grid. The drawing process follows the exponential distribution, and the scale parameter of the distribution, ψ_{μ}, is calibrated to match the mean retraining rate from the NLSY79.

The unemployment insurance benefit b is chosen so that it replaces about 40% of prior earnings. The income tax rate, τ, is set to the value that makes the government's budget balance. The benefit expiration rate, χ, is chosen so that the expected duration of eligibility is approximately 26 weeks. Home production, b_{min}, when the benefit is not available, is set to the value that prevents negative consumption.

The occupation and skill-specific production, y_{js}, is calibrated to match the college premium in each occupation and the non-routine cognitive premium among each education group. The premiums are obtained from the CPS.

To assign values to δ_{js}, the occupation and skill-specific job separation rate, and κ_{js}, the occupation and skill-specific job posting cost, I calculated the gross worker flows from the CPS using its panel structure. The CPS surveys the same household 4 months consecutively, skip 8 months, and then re-surveys for another 4 months. I restricted the sample to the households that are surveyed for the first time and the households that just come back to the survey after the break so that I can observe their employment status three months later. I calculated the quarterly job separation rate in occupation-j for skill-s workers as the fraction of employed skill-s workers in occupation-j who became unemployed three month later. I assign these values to δ_{js} in the model. Similarly, I calculated the quarterly job finding rate in occupation-j for skill-s workers as the fraction of unemployed skill-s workers who previously held a occupation-j job and became employed three months later. κ_{js} is calibrated to match these values.

Retraining takes 9 model periods (3 years assuming participants spend 3 quarters per year at school). The dropout rate, λ, is chosen to match the retraining completion rate from the NLSY97. The tuition, ν, is chosen to match the tuition in the data as a ratio of the average wages. To calculate the tuition-income ratio, I use in-state tuition data from the National Center for Education Statistics (NCES) and the average annual income among high-school graduates from the CPS. The tuition subsidy, ρ, is set to zero in the benchmark calibration since public-sponsored retraining programs are very limited.[5] In the later part of the paper, I adjust this parameter to examine the effects of subsidizing retraining.

[5] The Workforce Investment Act was introduced in 1998. Public-sponsored retraining before 1998 was provided through Job Training Partnership Act, which focused more on supporting the economically disadvantaged than retraining unemployed workers (Jacobson et al., 2005b). Even with WIA, public-sponsored retraining is limited. The sequential nature of the program may mean that not many unemployed workers never reach the training level of services(Frank and Minoff, 2005).

Table 6: Targeted moments

	Model	Target	Source
Skill premium in occupation-CG	28.21%	28.42%	CPS (1983-89)
Skill premium in occupation-R	25.60%	25.80%	CPS (1983-89)
occupation premium for skill-h	23.22%	19.37%	CPS (1983-89)
occupation premium for skill-l	20.72%	19%	CPS (1983-89)
Job finding rates at occupation-R for skill-l	44.31 %	45.14%	CPS (1983-89)
Job finding rates at occupation-R for skill-h	48.64%	48.94%	CPS (1983-89)
Job finding rates at occupation-CG for skill-l	47.97%	48.69%	CPS (1983-89)
Job finding rates at occupation-CG for skill-h	45.22%	44.93%	CPS (1983-89)
Tuition-income ratio	9.97%	10.81%	NCES, CPS (1983-89)
UI benefit-income ratio	39.88%	40%	Standard
Retraining population (23-33)	1.66%	1.66%	NLSY79

I use a constant returns to scale matching function that yields well-defined probabilities following Schaal (2012):

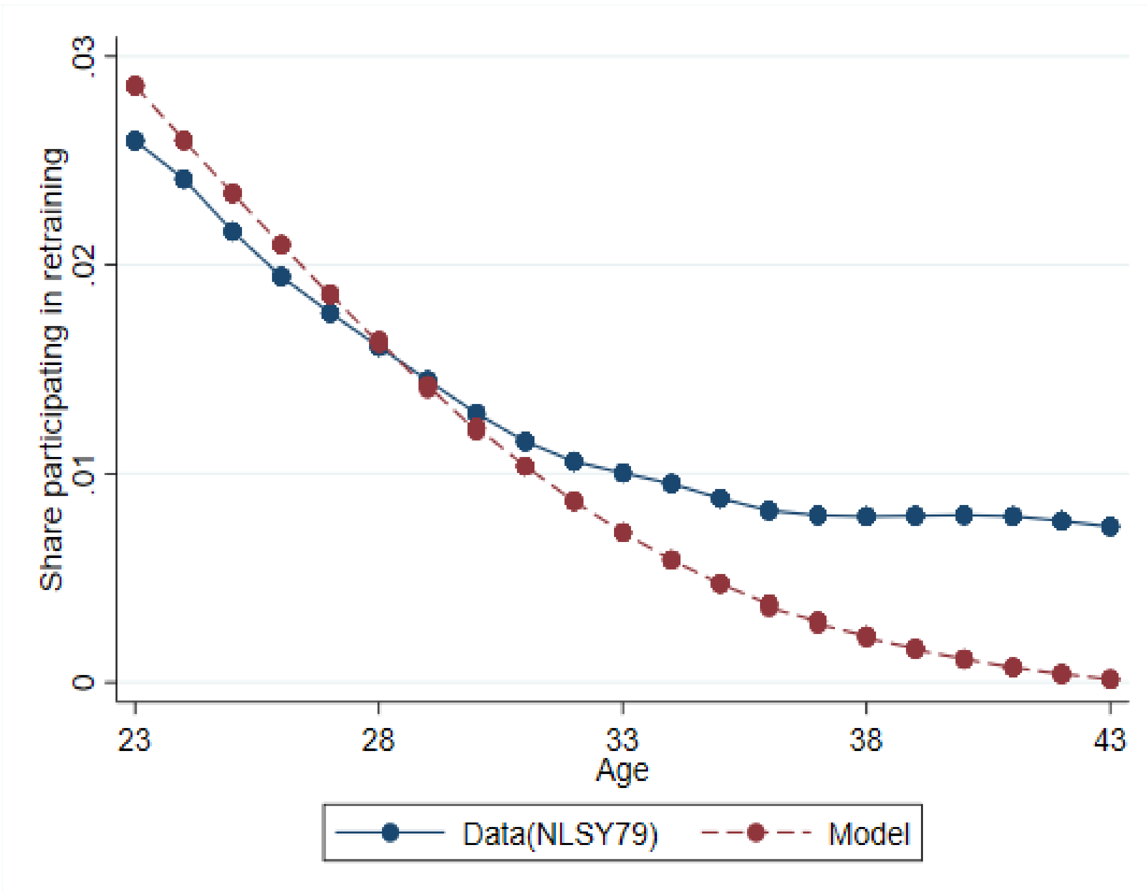
$$M(u,v) = \frac{uv}{(v^{\zeta} + u^{\zeta})^{\frac{1}{\zeta}}} \tag{11}$$

The firms' hiring rates are given by $q(\theta_t(s,j,\mu)) = \frac{M(u_t(t,s,j,\mu),v_t(t,s,j,\mu))}{v_t(t,s,j,\mu)}$, and the workers' job finding rates are given by $m(\theta_t(t,s,j,\mu)) = \frac{M(u_t(t,s,j,\mu),v_t(t,s,j,\mu))}{u_t(t,s,j,\mu)}$. The matching elasticity, ζ, is set to 0.5 as in Shi (2016).

4.2 Model Performance

Table 6 compares targeted moments between the model and the data. The statistics generated by the model are very close to those obtained from the data. Figure 10 graphically shows the model prediction of retraining rates by age. The model replicates retraining rates in the NLSY79 well for younger population but underpredicts retraining rates for older population. One possible explanation is that the model only takes account of economic aspects of retraining whereas in reality, people decide to go back to school for other reasons such as in search for a sense of accomplishment or the pure joy of learning. Such non-economic motivations can play a more important role in explaining older workers' retraining participation because a college degree may not be worthwhile for them in terms of career advancement.

Figure 10: Model fit



In the empirical analysis section, I documented that the NLSY97 cohorts have a considerably higher retraining rate than the NLSY79 cohorts. As a further test of the calibration, I see if the model can replicate this. I compare retraining rates of two groups of workers who face different labor markets in terms of wage premium and job transition rates. One group is thrown into a similar labor market that the NLSY79 cohorts (born 1957-1964) experienced when they were young workers. The other group is given the labor market conditions that the NLSY97 cohorts (born 1980-1984) faced early in their career.

The NLSY79 and NLSY97 cohorts have quite different labor market experiences. Table 7 compares some of the labor market characteristics that the two cohorts faced. The younger cohorts enjoyed a higher college premium and a higher non-routine cognitive occupation premium. However, they also suffered a worse labor market, featured as a lower job finding rate and a higher job separation rate. This is more prominent among low-skill workers. Some of it has to do with the fact that the economy has not yet fully recovered from the great recession when the younger cohort started their career. However, the fact that low-skill workers suffered a bigger drop in the job finding rate and a bigger rise in the job separation rate than high-skill workers reflects the gradual decline of routine jobs in the U.S. caused by automation and international trade.

The benchmark calibration features the labor market for the older cohort. Starting from there, I generate changes in the labor market characteristics that I observed in the data and see the resulting effects on retraining. I adjust matching quality differences across skill-occupation pairs (y_{js}) to reflect the rise of the skill and cognitive occupation premium. The substantial growth of these premiums increases benefits of retraining. In addition, I change parameters associated with job transition rates to match disproportionate changes in job finding and separation rates across skill-occupation pairs. Specifically, I adjust δ_{js} to match occupation and skill-specific job separation rates. I then vary κ_{js}, the vacancy posting cost, to match occupation and skill-specific job finding rates. Compared to the older cohort, the younger cohort, notably those without college education, faced a lower job finding rate and a higher job separation rate. These changes in job transition rates give low-skill workers another reason to retrain: career prospects without retraining look dim. I also vary tuition (ν) and the fraction of workers born

Table 7: Comparison of labor market characteristics between two cohorts

	1957-1964 cohorts	1980-1984 cohorts
College premium (occupation-CG)	28.42%	43.29%
College premium (occupation-R)	25.80%	36.09%
occupation premium (skill-h)	19.37%	30.78%
occupation premium (skill-l)	19%	24.38%
Job finding rates (occupation-CG & skill-h)	44.93%	35.28%
Job finding rates (occupation-CG & skill-l)	48.69%	28.49%
Job finding rates (occupation-R & skill-h)	48.93%	35.92%
Job finding rates (occupation-R & skill-l)	45.14%	28.15%
Job separation rates (occupation-CG & skill-h)	2%	2.9%
Job separation rates (occupation-CG & skill-l)	3.4%	4.68%
Job separation rates (occupation-R & skill-h)	3.4%	4.28%
Job separation rates (occupation-R & skill-l)	6.1%	6.89%
Tuition/Income	10.81%	27.02%
Share of college graduates at age 23	25.08%	36.65%

Note: Tuition includes tuition and required fee, averaged between four-year and two-year colleges.
Income is the average annual income among high-school graduates.
Source: CPS

as high-skill (M_h). It is well known that college tuition in the U.S. has risen significantly, and this increases the cost of retraining. I change the initial skill distribution as well to reflect the fact that the number of individuals who go straight to college from high school is higher among the younger cohort.

In the data I observe a higher retraining rate for the younger cohort than for the older cohort. Retraining rates among the younger cohort is about 6.52 percent compared to 1.66 percent among the older cohort. Adjusting for the labor-market related parameters mentioned above, the model yields retraining rates of 5.51 percent, predicting about 79 percent of the difference in retraining participation between the two cohorts observed in the data.

To further investigate the sources of higher participation in retraining among the younger cohort, I decompose the difference in retraining rates between the two cohorts into the contributions of each change in the labor market. To this end, I adjust one set of parameters at a time. For example, to see the contribution of wage premium, I adjust y_{js} to the values associated with the younger cohort with the rest of the parameters fixed at the level associated with the older cohort. Tuition and initial skill distribution are fixed at the level of the younger cohort.

Table 8: Retraining rates, in the data and as predicted by the calibrated model

	Retraining population (%)		
	1957-1964 cohorts	1980-1984 cohorts	Differences(pp)
Data	1.66	6.52	4.86
Model: all	1.66	5.51	3.85
Model: skill and occupation premium only	1.66	3.11	1.44
Model: job finding rates (High-skill) only	1.66	0	-1.66
Model: job finding rates (Low-skill) only	1.66	6.56	4.9
Model: separation rates (High-skill) only	1.66	0	-1.66
Model: separation rates (Low-skill) only	1.66	0.34	-1.32

Note: Tuition and the fraction born as high-skill are set to match the level for the younger cohort.

Table 8 presents the results. The decrease of job finding rates for low-skill workers cause the largest rise in retraining. The increase of skill and occupation premium generates the second-largest rise. These results suggest that low-skill workers retrain not only to get paid more but also to escape from the occupation in decline. There are factors that curb retraining as well. The increase of job separation rates for low-skill workers causes a modest decline in retraining. The decrease of job finding rates and the increase of job separation rates for high-skill workers decrease retraining by reducing the benefit of being high-skilled.

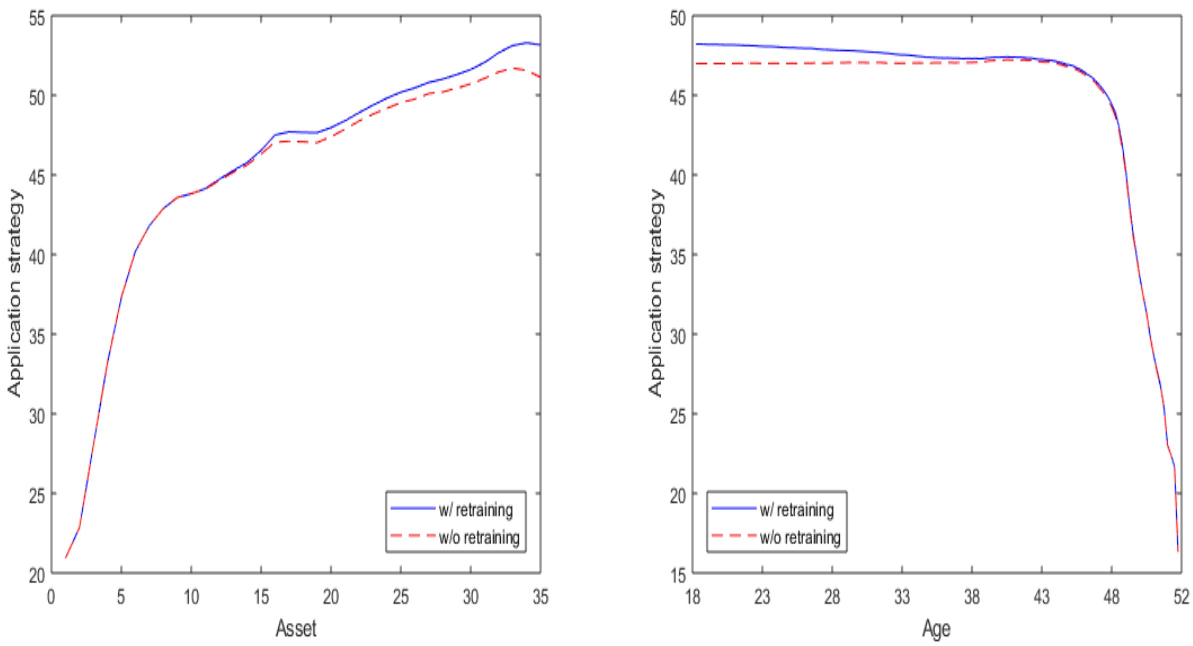
4.3 Counterfactual

In this section, I examine the aggregate effects of retraining on economy. I compare the benchmark economy to a counterfactual economy where retraining is not possible and see how retraining affects wage inequality and welfare.

4.3.1 Retraining and wage inequality

First, I look into the relationship between retraining and wage inequality. Retraining affects wage inequality by changing workers' optimal search strategies. In my model, unemployed workers face a trade-off between high wages and high job finding rates when they decide which jobs to apply for. In general, wealthy workers apply for high-paying but hard to obtain jobs since their wealth allows them to endure a longer unemployment duration. Retraining intervenes in this process by increasing the value of unemployment. The possibility of retraining makes unemployment less painful by giving unemployed workers one more option. This enables

Figure 11: Job search strategies



Note: This figure plots low-skill unemployed workers' optimal job search strategies by asset holdings (left) and by age (right). Higher numbers in the vertical axis represent higher-paying, harder to obtain jobs. The solid line shows job search strategies of the model with retraining. The dashed line shows job search strategies of the model without retraining.

unemployed workers to make bolder choices when they apply for jobs. As a result, unemployed workers apply for higher-paying jobs at a given amount of assets. Figure 11 presents low-skill unemployed workers' job application strategies by assets. The vertical axis represents job quality. The higher the number, the higher the wage and the lower the job finding rate. It shows that there exists a positive correlation between asset holdings and job quality, consistent with Eeckhout and Sepahsalari (2014) and Chaumont and Shi (2017). It also shows that, at a given level of assets, workers apply for better jobs in the benchmark economy where retraining is possible than they do in the counterfactual economy where there is no retraining.

This interaction between retraining and directed job search affects unemployed workers' re-employment wages. Table 9 compares the mean model wage among three different economies: the benchmark economy, a counterfactual economy where retraining doesn't exist, and another counterfactual economy where retraining completion rate is higher than it is in the benchmark

Table 9: Model predicted mean wages

	Model w/ retraining	Model w/ retraining	Model w/o retraining
	$\lambda = 0.04$	$\lambda = 0.08$	
High-skill	1.1261	1.1256	1.1253
Low-skill	0.8481	0.8025	0.7987
All	1.0042	0.8979	0.8844

Note: This table reports the model predicted mean wages for high- and low-skill workers. λ is the dropout rate.

economy. This economy can be considered as an economy that has a more effective retraining system than the benchmark economy. It has the highest retraining rates among the three economies.

Compared to the economy without retraining, the mean wage of low-skill workers is 0.5% higher in the benchmark economy and 6% higher in the economy with a high completion rate. This result is consistent with the mechanism explained above. Workers go for higher-paying jobs when there is a retraining channel, and therefore, get paid better. The mean wage of high-skill workers shows a similar pattern. The mean wage of high-skill workers is 0.03% higher in the benchmark economy and 0.08% higher in the economy with a high completion rate compared to the no-retraining economy. Even though high-skill workers are not directly affected by retraining rates, their wages are indirectly affected through the number of high-skill workers in the economy and corresponding income tax revenue. As fewer workers participate in retraining, fewer high-skill workers are created, and therefore, the income tax revenue decreases. Consequently, the income tax rate goes up, and after-tax wages decrease for high-skill workers. Although the mean wage increases for both skills, it increases more among low-skill workers, making the wage-gap between low- and high-skill workers shrink. The high-skill premium decreases from 40.9% in the no-retraining economy to 40.3% in the benchmark economy and to 32.8% in the economy with a high completion rate.

On the other hand, retraining makes the wage distribution within each skill group more dispersed. Figure 12 plots age-inequality profiles by education level. The wage standard deviation predicted from the model is smaller than that from the data, mainly because the model lacks employment-to-employment transition resulting in less variability in employment history.

Table 10: Model predicted st.d. of wages

	Model w/ retraining	Model w/ retraining	Model w/o retraining
	$\lambda = 0.04$	$\lambda = 0.08$	
High-skill	0.1203	0.1202	0.1202
Low-skill	0.0995	0.0926	0.0912
All	0.1736	0.18	0.1760

Note: This table reports the model predicted mean wages for high- and low-skill workers. λ is the dropout rate.

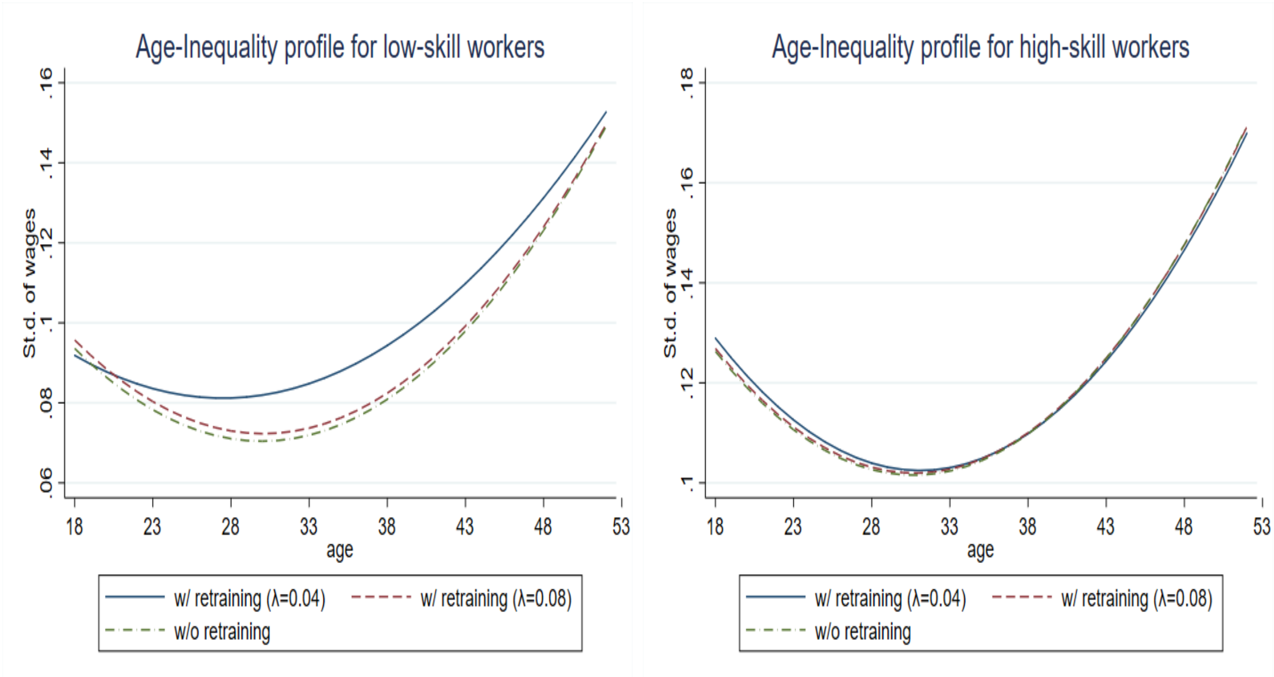
However, the model replicates the U-shape in the data well. The wage standard deviation is high among young workers. As they accumulate assets and gradually move to high-paying jobs, their wages converge. This leads to the initial reduction in the wage standard deviation. As workers get older, they start to have very different employment histories, which leads to the rise in the wage standard deviation. Table 10 reports the standard deviation of wages of the three economies. Compared to the no-retraining economy, the wage standard deviation is 0.08% higher for high-skill workers and 9.1% higher for low-skill workers in the economy with a high completion rate. The wage standard deviation among high-skill workers is higher in the benchmark economy because of newly-created high-skill workers. High-skill workers who just finished retraining tend to own lower levels of assets since they ran down their savings while retraining. To avoid extended unemployment, they apply for low-paying, easily attainable jobs, stretching the left end of the wage distribution. Similarly, the wage variance among low-skill workers is higher in the benchmark economy since retraining participants who drop out are likely to end up at the lower tail of the wage distribution.

In summary, as more workers participate in retraining, between-skill inequality decreases, and within-skill inequality increases.

4.3.2 Retraining and welfare

In this section, I examine the effects of retraining on workers' welfare. I assume the agents in the benchmark economy are transferred to a counterfactual economy where retraining is not possible. Then I calculate consumption equivalent, the remaining lifetime consumption that makes agents indifferent between the two economies. Everything else is the same between the

Figure 12: Age-Inequality profiles by skill



Note: This figure plots the standard deviation of wages by age. The left figure is for low-skill workers, and the right figure is for high-skill workers.

two economies except the income tax rate. The income tax rate is higher in the counterfactual economy. The lack of retraining in the counterfactual economy leads to a smaller tax revenue because fewer high-skill workers are created. To keep the government budget balanced, the income tax rate should rise by about 4.5 percent.

The results of the welfare analysis is given in table 11. Moving to the economy without retraining decreases welfare by about 1.5 percent of consumption on average. All workers in the benchmark economy are worse-off. For high-skill workers, welfare losses come exclusively from income tax increases. For low-skill workers, on the contrary, the losses come from several other sources as well as income tax increase. First, eliminating retraining alters workers' job application strategies. As I discussed in the previous section, with a lack of retraining, unemployed workers would rather go for low-paying, easily attainable jobs. This decreases their re-employment wages but increases their chances of finding a job. The effects on welfare are ambiguous. Second, in the counterfactual economy, workers tend to save less because they do not have to save money to retrain in the future, and also they do not have to hold as much precautionary savings as they face shorter unemployment duration by making safe application choices. This channel can have positive effects on welfare by increasing consumption. The last source of welfare changes is their lost opportunities to upgrade skills.

To understand the direction and magnitude of each effect, I decompose the welfare changes from eliminating retraining according to the channels suggested above. To this end, I block each channel in turn and calculate the welfare changes again. Specifically, I assume that the policy functions or parameters associated with the channel in interest are fixed at their benchmark level and re-calculate the value functions in the counterfactual economy. For instance, to see the effects coming through workers' optimal search strategies, I assume a worker in the counterfactual economy chooses the same firm he would have chosen in the benchmark economy. Column 2 in Table 12 shows welfare changes for low-skill workers with the tax effects excluded. The average welfare losses increased from -1.078 to -0.761 percent. Column 3 in Table 12 reports the results when the firm choice effects are excluded. Column 4 in Table 12 shows the case where the saving effects are removed. The average welfare decreases even more when the search strategy effects and the saving effects are not taken account of, implying that these two channels offset some of the losses from losing retraining. Overall, the tax, search strategy, and saving effects together account for about 25 percent of the total welfare losses. The rest comes from lost opportunities to upgrade skills. The contribution of each channel is different according to workers' employment status. Changes in income tax, optimal search strategies, and optimal savings explain around 35 percent and 42 percent of the welfare losses for the employed and non-participants, respectively. However, they barely explain the welfare losses of retraining participants, suggesting most of their losses come from their lost chances to upgrade skill. It is not surprising since they are the most likely to become high-skill workers.

4.4 Policy Implications

I now turn my attention to policy analysis. Government policies in the benchmark economy resembles unemployment policies of the US; they are more focused on passive labor-market policies such as insurance benefit rather than active market policies such as retraining. In

Table 11: Welfare changes

Age	Asset	Low-skill			High-skill	
		Employed	Unemployed		Employed	Unemployed
			Non-participants	Participants		
18-22	1st quartile	-0.0143	-0.0047	-0.0022	-0.0247	-0.0072
	2nd quartile	-0.0267	-0.0052	-0.0018	-0.0023	-0.0067
	3rd quartile	-0.0190	-0.0011	-0.0079	-3.1389e-04	-0.0025
	4th quartile	-0.1476	-0.0011	-0.0370	-1.8505e-04	-0.0048
23-27	1st quartile	-0.0053	-0.0018	-0.0032	-0.0014	-0.0028
	2nd quartile	-0.0291	-0.0014	-0.0025	-5.6435e-04	-0.0130
	3rd quartile	-0.0027	-3.4600e-04	-0.0143	-6.5554e-04	-0.0022
	4th quartile	-0.1840	-9.5811e-04	-0.1142	-4.0890e-04	-0.0084
28-32	1st quartile	-0.0049	-0.0017	-0.0015	-8.4848e-04	-0.0021
	2nd quartile	-0.0281	-0.0012	-0.0013	-0.0063	-0.0113
	3rd quartile	-4.4161e-04	-7.6116e-05	-0.0072	-7.9607e-04	-0.0025
	4th quartile	-0.0760	-3.5574e-04	-0.0559	-5.3898e-04	-0.0139
33-37	1st quartile	-0.0048	-0.0017	-4.5502e-04	-5.3898e-04	-0.0020
	2nd quartile	-0.0276	-0.0012	-4.3900e-04	-3.5669e-04	-0.0227
	3rd quartile	-1.2494e-04	-1.3804e-05	-0.0029	-6.6452e-04	-0.0028
	4th quartile	-0.0200	-6.5512e-05	-0.0197	-0.0026	-0.0345
38-42	1st quartile	-0.0047	-0.0017	-2.3178e-05	-7.9252e-04	-0.0027
	2nd quartile	-0.0231	-0.0012	-5.5142e-05	-1.6039e-04	-0.0044
	3rd quartile	-0.0029	-1.9182e-04	-6.4491e-04	-1.6283e-04	-0.0023
	4th quartile	-0.0101	-1.3065e-04	-0.0045	-0.0049	-0.0518
43-47	1st quartile	-0.0024	-7.7524e-04	-0.001	-7.0516e-05	-0.0011
	2nd quartile	-0.0105	-8.9856e-04	0.000	-5.7326e-05	-0.0017
	3rd quartile	-0.0052	-5.9124e-04	0.000	-5.0750e-04	-0.0044
	4th quartile	-0.0522	-8.0435e-04	-2.7976e-05	-0.0079	-0.1353
48-52	1st quartile	-1.7700e-04	-4.5964e-05	-2.6039e-04	-5.0107e-06	-2.7928e-05
	2nd quartile	-3.9793e-04	-3.3950e-05	0.000	-3.6889e-06	-5.9363e-05
	3rd quartile	-0.0136	-3.7167e-04	0.000	-6.7562e-05	-0.0046
	4th quartile	-0.0535	-6.1823e-04	0.000	-6.0750e-04	-0.0210
	Overall	-0.7694	-0.0302	-0.2780	-0.0593	-0.3689
				-1.506		

Note: This table presents welfare changes from getting rid of retraining by age, asset, employment status, and education level.
Results reported as change(%) in the remaining lifetime consumption relative to the benchmark economy.

Table 12: Welfare changes for low-skill workers (Decomposed)

	Scenario 1	Scenario 2	Scenario 3	Scenario 4
Employed	-0.769	-0.475	-0.779	-0.786
Non-participants	-0.030	-0.009	-0.035	-0.034
Participants	-0.278	-0.277	-0.283	-0.281
All	-1.078	-0.761	-1.097	-1.101

Note: Scenario 1: All effects considered, i.e., No decomposition.
Scenario 2: Tax effects excluded.
Scenario 3: Firm choice effects excluded.
Scenario 4: Saving effects excluded.
The detailed welfare over asset and age can be found in Table B3-B5.

this section, I use my model of retraining to simulate alternative policy scenarios where the government is more actively involved in retraining and explore their macroeconomic effects on the economy.

I compare five policies, all of which aim to encourage retraining among low-skill unemployed workers. The policy alternatives I consider are as follows. (1) government pays all the retraining costs. (2) retraining participants can receive unemployment benefit for a longer period of time than non-participants. (3) retraining participants can receive higher unemployment benefit than non-participants.[6] (4) there is no unemployment insurance benefit, and retraining costs are fully covered by government. (5) government pays retraining costs only for selected population (older and/or low-asset) who are the most reluctant to retrain. These polices are compared to the benchmark economy where unemployed workers retrain at their own expenses. Under all policies, the government budget is balanced.

Table 13 presents the main aggregate statistics in the steady states of economies implementing different policies in comparison to the benchmark economy. I find that universal free retraining results in the highest retraining participation for unemployed workers. However, it comes with a cost of high taxes; about 27% increase in taxes on labor income is needed to guarantee free training for all participants. From the perspective of cost effectiveness, combining retraining participation with higher insurance benefit yields the best outcome. It achieves an increase of 1.43 percentage point in retraining. The income tax rate decreases by 9.17 percent. It is the policy that maximizes the average welfare as well. It increases the average welfare by

[6] Policies (2) and (3) are inspired by the German system, as discussed in (Nie, 2010).

3.11 percent.

Although the average welfare of both high- and low-skill workers is the highest under the policy where retraining participants can receive higher unemployment insurance benefit, the welfare ordering of policies is not the same between the two skill groups. Since high-skill workers only care about the tax burden they are going to carry, they prefer policies that yield smaller tax increase than others, whereas low-skill workers consider the benefits and costs of retraining as well as income tax. For instance, low-skill workers will choose free retraining with no insurance benefit over combining retraining participation with longer duration of benefit receipt even though it comes with higher income tax.

One thing I want to point out here is that all of the policies suggested above would be more effective if they were implemented along with actions that improve retraining completion rates. Some of above polices yield considerable tax increases mainly because not many retraining participants translate into high-skill workers, only increasing the number of new taxpayers by so much. With a higher completion rate, increased government spending will be partly offset by increased tax revenue, and therefore the tax increase will not be as large. Since financial difficulties are one of the most common reasons of discontinuing college education, it is true that above policies can affect participants' decisions to drop out. Unfortunately, my model is not able to capture that since dropping out is considered as an exogenous shock. It will be an interesting extension to allow retraining participants to decide whether to continue retraining.

5 Conclusion

In this paper, I develop an overlapping-generations model featuring retraining and directed job search to study the macroeconomic effects of retraining. Low-skill unemployed workers in the model either search for jobs or participate in retraining. Retraining is stochastic. Conditional on successfully completing retraining, participants can get better-paying, more highly-skilled jobs. Non-participants decide which job to apply for by comparing wages against job finding rates. Wealthy workers apply for high-paying but hard to obtain jobs since they can survive

Table 13: Comparison of unemployment policies

	Policy 1	Policy 2	Policy 3	Policy 4	Policy 5
Tax	27.22	-6.42	-9.17	1.33	9.50
Retraining Rate	5.44	1.43	1.43	5.41	3.33
Between-skill Inequality	-3.75	-1.18	-1.63	-4.27	-3.27
Within-skill Inequality (High)	2.14	0.58	0.58	6.41	7.12
Within-skill Inequality (Low)	6.77	3.34	3.11	8.01	7.54
Welfare (Overall)	-5.02	2.41	3.11	-1.87	0.82
Welfare (High)	-5.91	1.50	1.66	-2.80	-1.99
Welfare (Low)	0.89	0.90	1.45	0.94	0.82

Note: Policy 1: Government pays all the retraining costs.
Policy 2: Retraining participants can receive unemployment benefit for a longer period of time than non-participants (up to two years).
Policy 3: Retraining participants can receive higher unemployment benefit than non-participants
Policy 4: No UI benefit + free retraining
Policy 5: Government pays retraining costs only for selected population (older and/or low-assets)
Results reported as percent change (percentage point change in case of retraining rate) relative to the benchmark scenario. The detailed welfare over asset and age can be found in Table B6-B7.

long unemployment duration.

I use the model to examine the effect retraining has on wage inequality and welfare. Retraining affects wage inequality by changing unemployed workers' job search strategies. It increases the value of unemployment and makes unemployed workers seek higher-paying jobs at a given asset level. As a result, re-employment wages increase for low-skill workers, and the between-skill inequality reduces. Retraining also affects wage inequality indirectly through workers' wealth. Newly-created high-skill workers and retraining participants who don't finish retraining tend to hold a small amount of assets.Therefore, they go for low-paying but easily attainable jobs. The constant flow into the lower tail of the wage distribution increases the within-skill inequality.

Eliminating the retraining channel in the benchmark economy makes everyone worse off. It yields welfare losses equivalent of 1.5 percent decrease in consumption. The welfare losses come from income tax increases, changes in optimal firm choices, changes in saving, and lost opportunities to upgrade skills. The first three channels account for about 25 percent of the average welfare losses. But they don't explain much of welfare losses for retraining participants, implying their losses mainly come from the lost chances to become high-skill workers.

I use the model to evaluate labor-market policies that aim to encourage retraining participation. I compare changes in retraining rates, tax increase, and welfare across policies. I show that combining retraining with more generous unemployment insurance benefit is the best policy in terms of cost-effectiveness and welfare.

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Appendix

A Value functions for high-skill workers

A.1 Unemployed workers

$$
\begin{aligned}
U_t^h(a,b,j) = \max_{a'} u(c, L_u) + \chi\beta[\max_{\mu'} m(\theta_{t+1}(h,j,\mu'))E_{t+1}^h(a',\mu',j) \\
+ (1 - m(\theta_{t+1}(h,j,\mu')))U_{t+1}^l(a',\mu',j)] + (1-\chi)\beta[\max_{\mu'} m(\theta_{t+1}(h,j,\mu'))E_{t+1}^h(a',\mu',j) \\
+ (1 - m(\theta_{t+1}(h,j,\mu')))U_{t+1}^l(a',\mu',j)], \quad t \leq T
\end{aligned}
$$

$$U_{T+1}^h(a,b,j) = 0$$

$$\text{s.t} \qquad c + a' = (1+r)a + b \quad \text{and} \quad a' \geq \underline{a}$$

A.2 Employed workers

$$E_t^h(a,\mu,j) = \max_{a'} u(c, L_e) + \beta[\delta_j U_{t+1}^h(a',b,j) + (1-\delta_j)E_{t+1}^h(a',\mu,j)], \quad t \leq T$$

$$E_{T+1}^h(a,\mu,j) = 0$$

$$\text{s.t} \qquad c + a' = (1+r)a + (1-\tau)w(\theta_t(h,j,\mu)) \quad \text{and} \quad a' \geq \underline{a}$$

B Additional tables and figures

Table B1: Descriptive statistics

	All		Non participants		Participants		
Variable	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.	
Age	30.14	5.23	29.58	4.57	27.88	4.17	***
Percent male	46.94	0.50	47.32	0.50	21.43	0.41	***
Percent black	18.19	0.39	18.17	0.39	19.05	0.39	
Percent Hispanic	30.89	0.46	30.90	46.21	30.36	0.46	
Percent married	44.42	0.50	44.5	0.50	0.39	0.49	
Total real asset ($)	37791.74	116007.9	37916.3	116630	29514.05	61591.31	
Residual total asset($)	0.00001	112385.5	14.90	113031.9	-989.9683	54456.05	
Real hourly wages($)	12.58	29.62	12.58	29.59	12.92	31.40	
Residual wages($)	-6.57E-09	29.56	-0.02	29.54	1.28	31.20	
Percent non-routine cognitive	17.02	0.38	16.93	0.38	23.21	42.35	**
Percent non-routine manual	18.13	0.39	18.00	0.38	26.19	0.44	***
Percent routine cognitive	26.36	0.44	26.18	0.44	38.10	0.49	***
Percent routine manual	38.49	0.49	38.89	0.49	12.50	0.33	***
AFQT	37734.65	24876.11	37626.06	24849.87	44979.19	25625.52	***
N of Obs	11,332		11,164		168		

Note: *Significant at 10%, **Significant at 5%, ***Significant at 1%
Source: NLSY79.

Table B2: Occupation by sex (Participants vs. Non-participants)

A. Male	Non-participants	Participants	
Percent non-routine cognitive	12.99	27.78	***
Percent non-routine manual	12.44	11.11	
Percent routine cognitive	10.81	25.00	***
Percent routine manual	63.77	36.11	***
B. Female	**Non-participants**	**Participants**	
Percent non-routine cognitive	20.47	21.97	
Percent non-routine manual	23.01	30.30	**
Percent routine cognitive	40.00	41.67	
Percent routine manual	16.53	6.06	***

*significant at 10%, **significant at 5%, ***significant at 1%
The corresponding table for the NLSY79 can be found in the Appendix.
Source: NLSY79.

Table B3: Welfare changes for low-skill workers, tax effects excluded

Age	Asset	Low-skill		
		Employed	Unemployed	
			Participants	Non-participants
18-22	1st quartile	-0.006	-0.002	-0.001
	2nd quartile	-0.005	-0.002	-0.003
	3rd quartile	-0.019	-0.008	-0.001
	4th quartile	-0.147	-0.037	-0.001
23-27	1st quartile	-3.76E-04	-0.003	-4.52E-05
	2nd quartile	-7.43E-04	-0.003	-2.07E-04
	3rd quartile	-0.003	-0.014	-3.12E-04
	4th quartile	-0.183	-0.114	-0.001
28-32	1st quartile	-3.48E-05	-0.002	-4.70E-06
	2nd quartile	-1.22E-04	-0.001	-1.67E-05
	3rd quartile	-4.06E-04	-0.007	-5.73E-05
	4th quartile	-0.076	-0.056	-3.41E-04
33-37	1st quartile	-1.81E-05	-4.54E-04	-6.24E-06
	2nd quartile	-1.07E-04	-4.38E-04	-4.48E-06
	3rd quartile	-4.77E-06	-0.003	-3.01E-07
	4th quartile	-0.020	-0.020	-5.71E-05
38-42	1st quartile	-2.24E-05	-2.32E-05	-7.77E-06
	2nd quartile	-1.11E-04	-5.51E-05	-5.53E-06
	3rd quartile	-2.22E-05	-6.44E-04	-1.12E-06
	4th quartile	-0.005	-0.005	-4.51E-06
43-47	1st quartile	-1.50E-05	0.000	-5.37E-06
	2nd quartile	-6.58E-05	0.000	-5.91E-06
	3rd quartile	-6.28E-05	-2.79E-05	-5.25E-06
	4th quartile	-0.001	-2.60E-04	-1.16E-05
48-52	1st quartile	-8.84E-06	0.000	-7.95E-07
	2nd quartile	-7.17E-05	0.000	-1.18E-06
	3rd quartile	-0.005	0.000	-1.49E-04
	4th quartile	-0.005	0.000	-1.45E-04
Overall		-0.475	-0.277	-0.009

Note: This table presents detailed welfare analysis of scenario 2 in Table 12. Results reported as change(%) in the remaining lifetime consumption relative to the benchmark economy.

Table B4: Welfare changes for low-skill workers, firm choice effects excluded

Age	Asset	Low-skill		
		Employed	Unemployed	
			Participants	Non-participants
18-22	1st quartile	-0.018	-0.002	-0.006
	2nd quartile	-0.030	-0.002	-0.007
	3rd quartile	-0.023	-0.008	-0.002
	4th quartile	-0.158	-0.039	-0.002
23-27	1st quartile	-0.006	-0.003	-0.002
	2nd quartile	-0.030	-0.003	-0.002
	3rd quartile	-0.003	-0.014	-4.89E-04
	4th quartile	-0.190	-0.117	-0.001
28-32	1st quartile	-0.005	-0.002	-0.002
	2nd quartile	-0.028	-0.001	-0.001
	3rd quartile	-0.001	-0.007	-9.80E-05
	4th quartile	-0.078	-0.057	-4.21E-04
33-37	1st quartile	-0.005	-4.55E-04	-0.002
	2nd quartile	-0.028	-4.39E-04	-0.001
	3rd quartile	-1.25E-04	-0.003	-1.38E-05
	4th quartile	-0.020	-0.020	-6.63E-05
38-42	1st quartile	-0.005	-2.32E-05	-0.002
	2nd quartile	-0.023	-5.51E-05	-0.001
	3rd quartile	-0.003	-0.001	-1.91E-04
	4th quartile	-0.010	-0.005	-1.30E-04
43-47	1st quartile	-0.002	0.000	-0.001
	2nd quartile	-0.011	0.000	-0.001
	3rd quartile	-0.005	-2.80E-05	-0.001
	4th quartile	-0.033	-2.60E-04	-0.001
48-52	1st quartile	-1.76E-04	0.000	-4.58E-05
	2nd quartile	-3.96E-04	0.000	-3.38E-05
	3rd quartile	-0.014	0.000	-3.70E-04
	4th quartile	-0.053	0.000	-0.001
Overall		-0.779	-0.283	-0.035

Note: This table presents detailed welfare analysis of scenario 3 in Table 12. Results reported as change(%) in the remaining lifetime consumption relative to the benchmark economy.

Table B5: Welfare changes for low-skill workers, saving effects excluded

Age	Asset	Low-skill		
		Employed	Unemployed	
			Participants	Non-participants
18-22	1st quartile	-0.018	-0.002	-0.006
	2nd quartile	-0.029	-0.002	-0.007
	3rd quartile	-0.027	-0.008	-0.002
	4th quartile	-0.158	-0.039	-0.002
23-27	1st quartile	-0.006	-0.003	-0.002
	2nd quartile	-0.029	-0.003	-0.002
	3rd quartile	-0.004	-0.014	-4.74E-04
	4th quartile	-0.187	-0.116	-0.001
28-32	1st quartile	-0.005	-0.002	-0.002
	2nd quartile	-0.028	-0.001	-0.001
	3rd quartile	-0.001	-0.007	-9.65E-05
	4th quartile	-0.077	-0.056	-3.93E-04
33-37	1st quartile	-0.005	-4.55E-04	-0.002
	2nd quartile	-0.028	-4.39E-04	-0.001
	3rd quartile	0.000	-0.003	-1.38E-05
	4th quartile	-0.020	-0.020	-6.63E-05
38-42	1st quartile	-0.005	-2.32E-05	-0.002
	2nd quartile	-0.023	-5.51E-05	-0.001
	3rd quartile	-0.003	-0.001	-1.91E-04
	4th quartile	-0.017	-0.005	-1.30E-04
43-47	1st quartile	-0.002	0.00	-0.001
	2nd quartile	-0.011	0.00	-0.001
	3rd quartile	-0.005	-2.80E-05	-0.001
	4th quartile	-0.033	-2.60E-04	-0.001
48-52	1st quartile	-1.76E-04	0.00	-4.57E-05
	2nd quartile	-3.96E-04	0.00	-3.37E-05
	3rd quartile	-0.014	0.00	-3.70E-04
	4th quartile	-0.053	0.00	-0.001
Overall		-0.786	-0.281	-0.034

Note: This table presents detailed welfare analysis of scenario 4 in Table 12. Results reported as change(%) in the remaining lifetime consumption relative to the benchmark economy.

Table B6: Comparison in welfare across policies (Low-skill workers)

		Policy 1	Policy 2	Policy 3	Policy 4	Policy 5
18-22	1st quartile	0.1226	0.0776	0.0974	0.2455	0.0771
	2nd quartile	0.0998	0.0469	0.0506	0.1358	0.0522
	3rd quartile	0.0750	0.0335	0.0347	0.1040	0.0300
	4th quartile	0.1417	0.0547	0.0506	0.2029	0.0430
23-27	1st quartile	0.0524	0.0504	0.0683	0.1464	0.0579
	2nd quartile	0.0602	0.0306	0.0339	0.0832	0.0433
	3rd quartile	0.0601	0.0266	0.0293	0.0889	0.0360
	4th quartile	0.3294	0.1178	0.1108	0.4852	0.1279
28-32	1st quartile	-0.0019	0.0299	0.0453	0.0652	0.0294
	2nd quartile	0.0406	0.0218	0.0251	0.0587	0.0384
	3rd quartile	0.0403	0.0202	0.0235	0.0652	0.0364
	4th quartile	0.2116	0.0801	0.0815	0.3338	0.1733
33-37	1st quartile	-0.0370	0.0159	0.0278	0.0042	-0.0044
	2nd quartile	0.0187	0.0122	0.0157	0.0340	0.0257
	3rd quartile	0.0183	0.0124	0.0162	0.0389	0.0307
	4th quartile	0.0917	0.0403	0.0467	0.1815	0.1402
38-42	1st quartile	-0.0309	0.0078	0.0165	-0.0141	-0.0104
	2nd quartile	-0.0081	0.0043	0.0064	8.6706e-04	1.8953e-04
	3rd quartile	-0.0102	0.0048	0.0084	5.1245e-04	0.0018
	4th quartile	-0.0216	0.0123	0.0209	0.0196	0.0143
43-47	1st quartile	-0.0096	0.0022	0.0092	-0.0124	-0.0033
	2nd quartile	-0.0075	0.0018	0.0042	-0.0139	-0.0026
	3rd quartile	-0.0238	0.0056	0.0134	-0.0428	-0.0083
	4th quartile	-0.1314	0.0207	0.0481	-0.0506	-0.0322
48-52	1st quartile	-0.0023	5.2379e-04	0.005	-0.1299	-7.8855e-04
	2nd quartile	-0.0739	0.0273	0.4620	-0.6172	-0.0255
	3rd quartile	-0.0656	0.1300	0.0564	-0.3914	-0.0290
	4th quartile	-0.0476	0.0129	0.0308	-0.0840	-0.0237
	Overall	0.8913	0.9012	1.4545	0.9383	0.8174

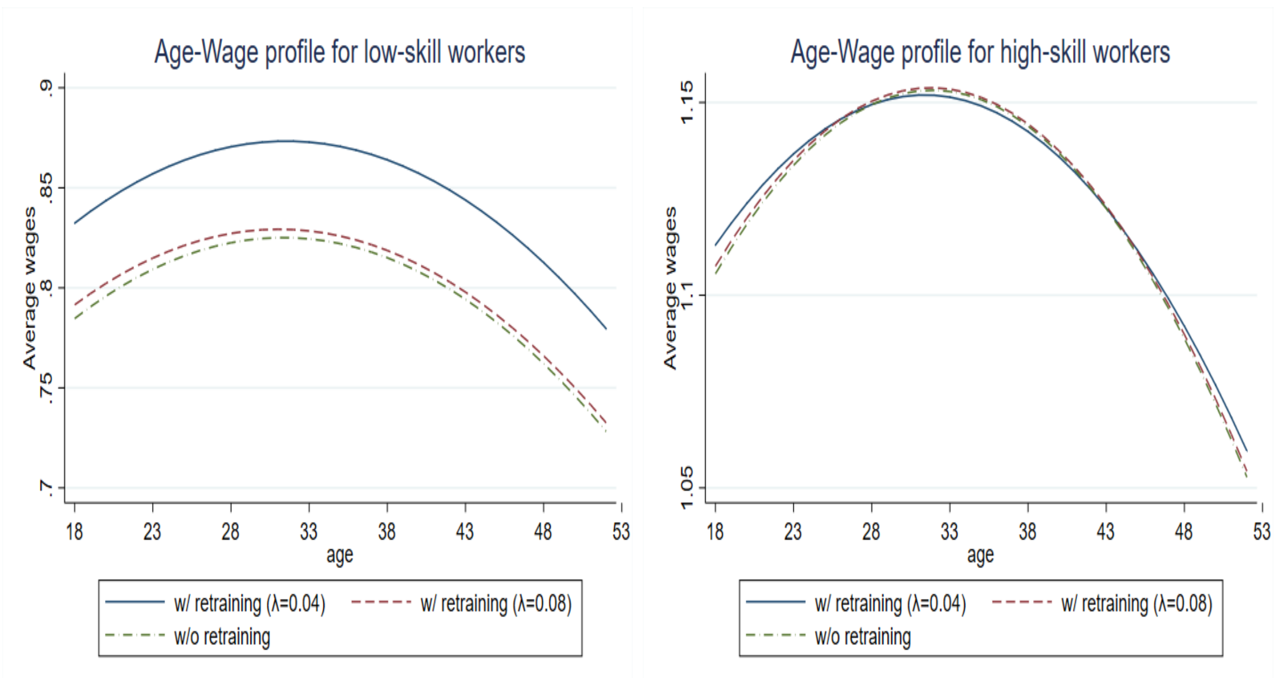
Note: Policy 1: Government pays all the retraining costs. Policy 2: Retraining participants can receive unemployment benefit for a longer period of time than non-participants (up to two years). Policy 3: Retraining participants can receive higher unemployment benefit than non-participants Policy 4: No UI benefit + free retraining Policy 5: Government pays retraining costs only for selected population (older and/or low-assets)
Results reported as percent change (percentage point change in case of retraining rate) relative to the benchmark scenario.

Table B7: Comparison in welfare across policies (High-skill workers)

		Policy 1	Policy 2	Policy 3	Policy 4	Policy 5
18-22	1st quartile	-0.4963	0.1457	0.1982	-0.0938	-0.2122
	2nd quartile	-0.4458	0.0743	0.1065	-0.0376	-0.1506
	3rd quartile	-0.0765	0.0166	0.0237	-0.0069	-0.0257
	4th quartile	-0.0015	3.2954e-04	4.7050e-04	-1.3884e-04	-5.0454e-04
23-27	1st quartile	-0.1270	0.0659	0.1617	-0.0277	-0.2651
	2nd quartile	-0.4365	0.0752	0.0866	-0.0266	-0.1244
	3rd quartile	-0.1367	0.0292	0.0416	-0.0123	-0.0455
	4th quartile	-0.0256	0.0057	0.0081	-0.0025	-0.0087
28-32	1st quartile	-0.0331	0.0095	0.0143	-0.0077	-0.0125
	2nd quartile	-1.1446	0.5707	0.1658	-0.0555	-0.1692
	3rd quartile	-0.2423	0.0375	0.0522	-0.0186	-0.0854
	4th quartile	-0.0629	0.0141	0.0202	-0.0067	-0.0215
33-37	1st quartile	-0.0214	0.0052	0.0075	-0.0053	-0.0076
	2nd quartile	-0.4652	0.0557	0.1077	-0.1901	-0.0957
	3rd quartile	-0.2579	0.0606	0.1123	-0.0658	-0.1305
	4th quartile	-0.0873	0.0197	0.0282	-0.0105	-0.0300
38-42	1st quartile	-0.0116	0.0028	0.0040	-0.0033	-0.0041
	2nd quartile	-0.1353	0.0491	0.0601	-0.0165	-0.1812
	3rd quartile	-0.5483	0.0722	0.0919	-0.0675	-0.1394
	4th quartile	-0.1109	0.0247	0.0353	-0.0162	-0.0377
43-47	1st quartile	-0.0036	8.5922e-04	0.0012	-0.0014	-0.0013
	2nd quartile	-0.0391	0.0096	0.0207	-0.0092	-0.0167
	3rd quartile	-0.2749	0.0415	0.0794	-0.0619	-0.0554
	4th quartile	-0.3345	0.0613	0.0950	-0.0746	-0.0642
48-52	1st quartile	-3.8490e-04	8.9205e-05	1.2788e-04	-0.0169	-1.3426e-04
	2nd quartile	-0.0351	0.0147	0.0534	-1.4407	-0.0113
	3rd quartile	-0.3295	0.0365	0.0756	-0.5124	-0.0877
	4th quartile	-0.0267	0.0055	0.0076	-0.0154	-0.0088
	Overall	-5.9104	1.5051	1.6595	-2.8038	-1.9930

Note: Policy 1: Government pays all the retraining costs. Policy 2: Retraining participants can receive unemployment benefit for a longer period of time than non-participants (up to two years). Policy 3: Retraining participants can receive higher unemployment benefit than non-participants Policy 4: No UI benefit + free retraining Policy 5: Government pays retraining costs only for selected population (older and/or low-assets)
Results reported as percent change (percentage point change in case of retraining rate) relative to the benchmark scenario.

Figure B1: Age-Wage profiles by skill



Note: This figure plots the mean wage by age. The left figure is for low-skill workers, and the right figure is for high-skill workers.

C Solution Algorithm

Starting at $t = T$ and working backwards, the solution is given as:

1. Compute the firm value function, $J_t(s, j, \mu)$.

2. Computer the market tightness, $\theta_t(s, j, \mu)$, by equation (9).

3. Guess a value for the income tax rate, τ.

4. Solve the employed high-skill worker problem for all t, a, μ, j and compute optimal savings $a^h_{t+1}(a_t, \mu, j)$.

5. Solve the unemployed high-skill worker problem for all t, a, b, j and compute optimal savings $a^h_{t+1}(a_t, b, j)$ and optimal firm choices $\theta^h_{t+1}(a_t, b, j)$.

6. Solve the employed low-skill worker problem for all t, a, μ, ψ, j and compute optimal savings $a^l_{t+1}(a_t, \mu, \psi, j)$.

7. Solve the low-skill non-participant problem for all t, a, μ, ψ, j and compute optimal savings $a^l_{t+1}(a_t, \mu, \psi, j)$ and optimal firm choices $\theta^l_{t+1}(a_t, b, \psi, j)$.

8. Solve the low-skill participant problem for all t, a, μ, ψ, j, z and compute optimal savings $a^l_{t+1}(a_t, \mu, \psi, j, z)$ and optimal firm choices $\theta^l_{t+1}(a_t, b, \psi, j, z)$.

9. Solve the retraining decision for low-skill unemployed workers and recover $D_t(a, b, \psi, j)$.

10. Using the policy functions, compute the distribution functions over the state variables.

8. Using the policy functions and distribution functions, compute the total tax revenue and government expenditure on unemployment insurance benefit. Check if the government budget is balanced.

If the government budget is balanced, the model is solved. If not, go back to 3 and adjust the tax rate.